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# THEMATICS 10

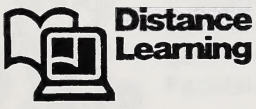
## LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

# Mathematics 10

# LEARNING FACILITATOR'S MANUAL



## Note

**This Mathematics 10 Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final test until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.**

| This document is intended for | |
|---|---|
| Students | |
| Teachers (Mathematics 10) | ✓ |
| Administrators | |
| Parents | |
| General Public | |
| Other | |

Mathematics 10
Learning Facilitator's Manual
Units 1-8
Alberta Distance Learning Centre
ISBN No. 0-7741-0744-8

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# Teachers

## Register with the Alberta Distance Learning Centre

The Alberta Distance Learning Centre is dedicated to upgrading and continually improving your Learning Facilitator's Manual so that it accurately reflects any necessary revisions we have had to make in the student module booklets, assignment booklets or the sample final test. The types of revisions that will be made are those that make the course more accurate, current, or more effective.

The ADLC will send you the latest enhancements and upgrades for your Learning Facilitator's Manual if you return the following registration card to: Alberta Distance Learning Centre, Box 4000, Barrhead, Alberta, T0G 2P0, Attention: Instructional Design and Development.

**ADLC Learning Facilitator's Manual**
**Registration Card**

| | |
|---|---|
| First Name | Surname |
| School Name | School Phone Number |
| School Address | |
| City | Postal Code |
| Course Title | Approximate Date of Purchase |

You can help ensure that distance learning courseware is of top quality by letting us know of areas that need to be adjusted. Call the Alberta Distance Learning Centre free of charge by using the RITE line and ask for the Editing Unit. Also, a teacher questionnaire has been included at the back of most Learning Facilitator's Manuals. Please take a moment to fill this out.

We look forward to hearing from you!

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Unit 7

Unit 8

Teacher's Copy of the Final Test with Marking Guide

Final Test

Teacher Questionnaire

# Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

## Which Category Do You Fit?

- ❑ Small Schools Teacher
  - ❑ inexperienced
  - ❑ experienced, but in other subject areas
  - ❑ experienced in teaching Mathematics 10, but wanting to try a different approach
- ❑ Distance Learning Teacher
  - ❑ travelling to schools within the jurisdiction
  - ❑ using facsimile and teleconferences to teach students within the area
- ❑ Larger Schools Teacher
  - ❑ inexperienced
  - ❑ experienced in teaching Mathematics 10, but wanting to try a different approach

Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situations.

### Advantages for Students

- incorporates a strong learner-centred philosophy
- promotes such qualities in the learner as autonomy, independence, and flexibility
- is developed through media which suit the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

### Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects the Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

This Learning Facilitator's Manual begins with an overview of the current Alberta Education Program of Studies for high school mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching high school mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new learning package and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The module booklets, assignment booklets, and LFMs are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.

## Overview of the Program of Studies

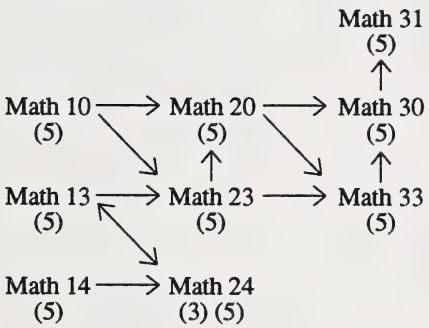
A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology, the emphasis will not be so much on memorizing formulas but on how to use mathematical information to solve problems. Students will be expected to construct, investigate, translate, discuss, analyse, and formulate mathematics in a dynamic way.

The Mathematics 10 – 20 – 30 – 31 sequence is designed for students with an interest and aptitude in mathematics who are intending to pursue postsecondary studies at a university or in a mathematics-intensive program at a technical school or college. Having successfully completed Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

The Mathematics 13 – 23 – 33 sequence is the mainstream mathematics program designed for students who require mathematics to prepare for universities, colleges, trades, and employment. The Mathematics 13 – 23 – 33 sequence will satisfy the requirements for the General High School Diploma.

The Mathematics 14 – 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses.

The complete high school mathematics program is shown along with recommended transfer points. The number in brackets indicates credits.




This mathematics course is divided into units. Each unit covers a major area of mathematical study and is divided further into specific content areas called topics. Each unit has its own assignment booklet which the student completes and submits at the end of that unit.

Some of the distance learning courses are divided into sections within modules. If you receive letters or other materials from the Alberta Distance Learning Centre which refer to modules, please remember that a module is the same as a unit, and a section is the same as a topic.

## Overview of Mathematics 10

Unit 1: Number Systems

Unit 2: Operations on Polynomials

Unit 3: Equations and Inequalities

Unit 4: Factoring Polynomials

Unit 5: Coordinate Geometry

Unit 6: Systems of Equations

Unit 7: Trigonometry

Unit 8: Statistics

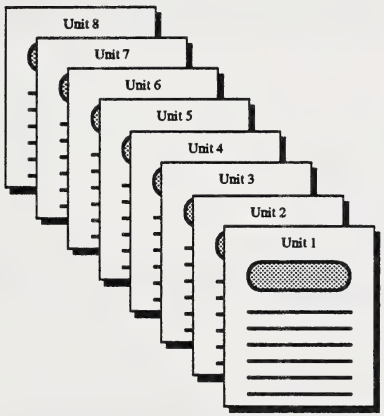
The Mathematics 10 course is the first course in the Mathematics 10 – 20 – 30 acedemic series. The course is divided into eight units as outlined. The course is offered for five credits only.

# Structure of the Learning Package

## Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

### Units




| Contents at a Glance |
|---|
| Unit Overview |
| What You Already Know |
| Review |
| Topic 1<br>Activity 1<br>Activity 2<br>etc. |
| Topic 2<br>Activity 1<br>Activity 2<br>etc. |
| Topic 3<br>Activity 1<br>Activity 2<br>etc. |
| Unit Summary |
| Appendices |

The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant, realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques enable learners to learn on their own for at least some of the time.

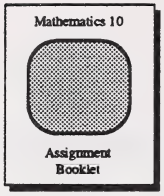
The structure of the unit booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The unit overview introduces the unit topic or theme. A graphic representation has been included to help visual learners and poor readers. The course overview states the weightings of each assignment.

The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centred around an objective.

The activities may involve print, audio, video, or computer formats. At times the student and the learning facilitator are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Enrichment are optional pathways. This flexibility caters to each student's personal situation.

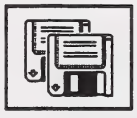
The summary focuses on the skills and strategies that the student has learned.

## Assignment Booklet




Accompanying each unit is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the unit materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

## Media



COMPUTER DISK

VIDEOCASSETTE

AUDIOCASSETTE

The package also includes references to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever video or computer software references have been included, a print pathway is also available. This way, if the media resource isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Special audiocassettes guide the student through a difficult concept. The appearance of the audiocassette icon reminds students that there is this additional help available.

If the students are working individually, you may find this cassette a valuable asset. If you are working in a large group, you may wish to guide the students yourself.

## Materials, Media, and Equipment

### Mandatory Components

| Equipment (Hardware) | Media | Materials |
|---|---|---|
| • audiocassette player | • prepared audiocassettes (come with learning package) | • LFM for Mathematics10<br>• one complete set of unit booklets (8) and assignment booklets (8) for each student<br>• geometry set<br>• scientific calculator or graphing calculator<br>• There is a final test. |

### Optional Components

| Equipment (Hardware) | Media | Materials |
|---|---|---|
| • VCR | • videocassettes<br>Videocassettes used in the course may be available from the Learning Resources Distributing Centre or ACCESS Network. You may also wish to call your regional library service for more information. | Mathematics Dictionary, 4th Edition Can Norstrand Reinhold, 1976<br>ISBN 0-442-24091-0<br>Holt Math 10, Rinehart & Winston of Canada, 1987<br>Mathematics: Principles and Process 10; Nelson Canada 1987 |
| • Computer | • computer disks<br>Check with local software outlets. | Mathematics 10 Addison-Wesley, 1987 |

# Using This Learning Package in the Classroom

## Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

## Open-Learning Classroom

Open learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners individually, regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. However, media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

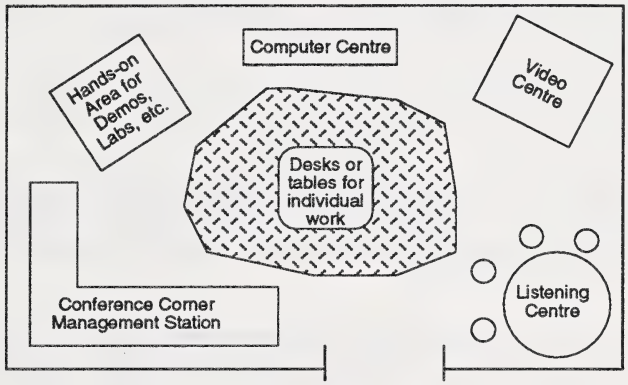
The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

### The Key to a Successful Open-Learning System

## Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media.




You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

## Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the unit booklet. High school students have solutions, models, explanations, and guides included in the appendix of every unit booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for students to do these activities carefully before attempting the assignments in the assignment booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a biweekly conference scheduled by you, or as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse, etc.), something prescheduled like the demonstration of a process, or something involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

### Management

Good management of an open-learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

- Scheduling, Distributing, and Managing Resources – As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.

- Scheduling Students – Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g., all morning), teachers should discourage this. Concentration, retention, and motivation are improved by taking scheduled breaks.

- Monitoring Student Progress – You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

### Sample of a Student Progress Chart

| Mathematics 10 | | Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Final Test |
|---|---|---|---|---|---|---|---|---|---|---|
| *Billy Adams* | P | | | | | | | | | |
| | A | | | | | | | | | |
| *Louise Despins* | P | | | | | | | | | |
| | A | | | | | | | | | |
| *Violet Klaissian* | P | | | | | | | | | |
| | A | | | | | | | | | |
| P = Projected Completion Date A = Actual Completion Date | | | | | | | | | | |

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- Recording Student Assessments – You will need to record the marks awarded to each student for work completed in each unit assignment booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

### Sample of a Student Assessment Chart

| Mathematics 10 | Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Year's Average | Final Test | Final Mark |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Billy Adams* | 67 | 65 | 54 | 47 | 78 | 67 | 63 | 77 | 65 | | |
| *Louise Despins* | 43 | 50 | 54 | 55 | 48 | 42 | 52 | 41 | 48 | | |
| *Violet Klaissian* | 65 | 65 | 66 | 68 | 67 | 70 | 66 | 61 | 66 | | |

Letter grading could easily be substituted.

- Recording Effectiveness of System – Keep ongoing records of how the system is working. This will help you in future planning.

### Sample of a System Assessment Chart

| Unit 1 | | | |
|---|---|---|---|
| Date | Unit Booklet | Assignment Booklet | Resources/Media |
| | | | |

### The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centred, not teacher-centred. The teacher needs to facilitate learning by providing general support to the learner.

## Evaluation

Evaluation is important to the development of every learner. Data gathering and processing, and decision making, at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially designed learning packages contain many kinds of informal and formal evaluation.

### Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

**Observation Checklist**

1. Does the student approach the work in a positive manner?
2. Is the student struggling with the reading level?
3. Does the student make good use of time?
4. Does the student apply an appropriate study method?
5. Can the student use references effectively, etc.?

Observation may suggest a need for an individual interview with a student.

## Individual Conferences

Individual conferences may be paced (scheduled) by the calendar, at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and can assess the student's attitudes toward the subject, the program, school, and self, as well as the student's relationship with other students. With guided questions the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular topic, unit, or subject area.

## Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming autonomous learners. By instructing, motivating, providing positive reinforcement, and systematically supervising, the learning facilitator will help students develop a positive attitude toward their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for the student. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment in your assignment booklet?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this unit?

A chart or checklist could be used for recording responses.

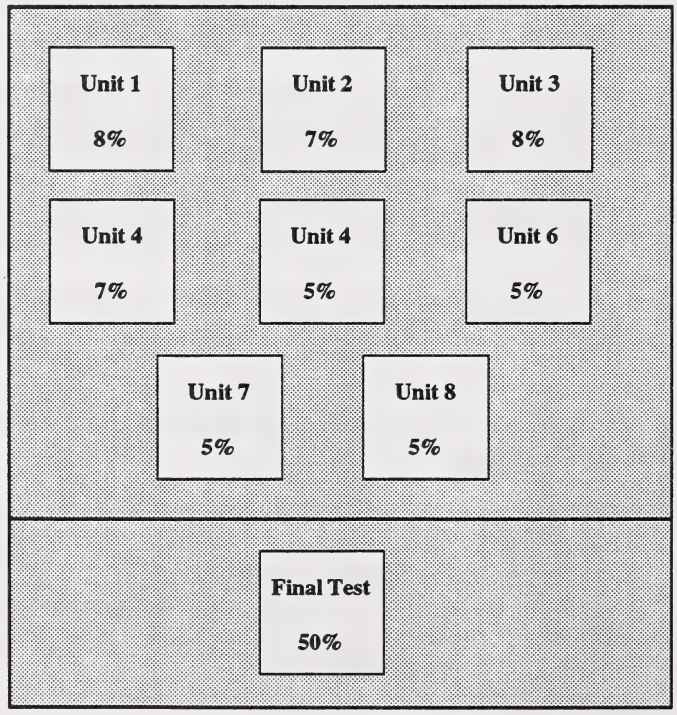
## Informal Evaluation: Assignments

Informal evaluation, such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behaviour and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the unit. **The student's mark for the unit may be based solely on the outcome of learning evident in the assignment booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated, and that all students clearly understand what is expected.

## Final Test

All LFMs include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, gives the teacher precise information concerning what each student can or cannot do. Answers and explanations and marking guides are also included. The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.



## Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the students and teacher should be analysed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

### Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

- understanding and using instructional materials (table of contents, index, list of illustrations, and appendices)
- interpreting graphs and charts
- using reference materials
- recognizing special symbols
- using a scientific or graphing calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

### Reading Level

These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks
- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts

- the recognition that vocabulary and concepts are basic to understanding content materials and, thus, must be handled systematically (defined in context, marginal notes, footnotes, and often in a specialized glossary)

- the acknowledgement that background knowledge and experience play a vital role in comprehension

- the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software as an alternative to print-based learning

- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

- the inclusion of media pathways and activity choices to encourage an active rather than passive approach

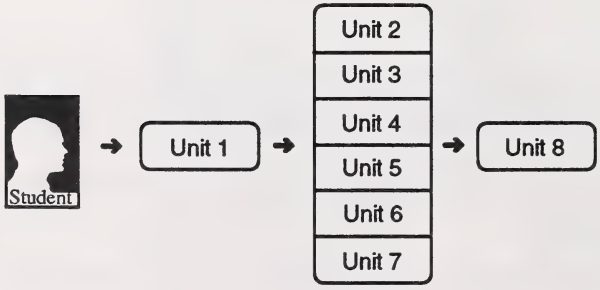
- instruction in a meaningful setting rather than in a contrived, workbook style

- using purposeful reading, viewing, and doing to produce better interpretation of the course materials

- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary, and extending (This structure closely resembles the reading process.)

To help make the learning package more readable, you can begin your unit preparation by reading (viewing, listening to) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic or by using visuals and guided questions to predict what the topic might be about.

It is recommended that you start with Unit 1 because this unit includes basic introductory information, and it is also recommended that you end with Unit 8.



# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL

## UNIT 1

## NUMBER SYSTEMS

Distance Learning

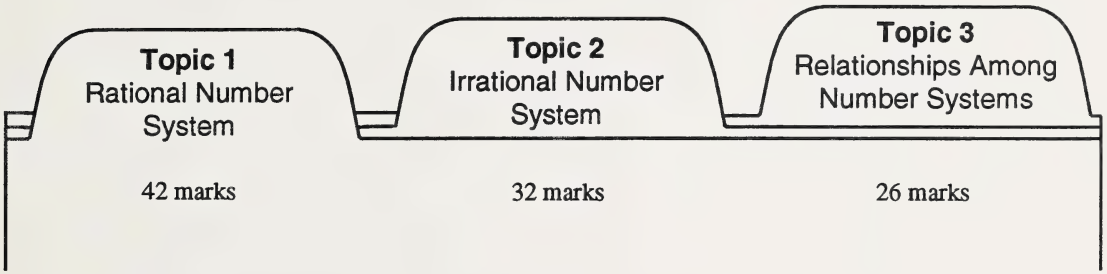
Alberta EDUCATION

## Unit 1 Overview

This unit called Number Systems is worth 10% of the course mark.

The unit consists of three topics. Each topic has an assignment in the Assignment Booklet.

### Unit 1 Number Systems




### Concept Introduction Possibilities

Show students how they may get results on their calculators that include more than the number of decimal places that normally show on the display. A method for approximating repeating decimal equivalents for fractions is shown in the Math 10, Teacher's Resource Manual, page 10.17. This is an interesting activity to show how the accuracy of the calculator may be extended. It also demonstrates that some operations which may not be immediately obvious can be carried out. (This method is also outlined in the Teacher's Resource Manual for *Holtmath 10*, Chapter 1, page 2.)

### Possible Presentation Activities

- use of simple Venn diagram
- use of a number line
- use of calculators, tables, charts, etc.
- use of squares and rulers

### Marking Guidelines

- If the answer is correct, give full marks whether or not work is shown.
- If the answer is incorrect and work is shown, give partial marks for correct work.

## Topic 1: Rational Number System

1. Evaluate the following.

(2) a. $4\times72\div12+(8-4)-39=4\times72\div12+4-39$
$=288\div12+4-39$
$=24+4-39$
$=-11$

(2) b. $[(6+8)\div7]\times2+3=(14\div7)\times2+3$
$=2\times2+3$
$=4+3$
$=7$

(2) c. $\dfrac{[19-(3\times6)]\times5}{2\times4-3\times3}=\dfrac{(19-18)\times5}{8-9}$
$=\dfrac{1\times5}{-1}$
$=-5$

2. Find the missing term for each fraction.

(2) a. $\dfrac{4}{5}=\dfrac{?}{35}$

$\dfrac{4}{5}\times\dfrac{7}{7}=\dfrac{\boxed{28}}{35}$

(2) b. $\dfrac{11}{13}=\dfrac{33}{?}$

$\dfrac{11}{13}\times\dfrac{3}{3}=\dfrac{33}{\boxed{39}}$

(2) c. $\dfrac{?}{8}=\dfrac{3}{4}$

$\dfrac{\boxed{6}}{8}=\dfrac{2}{2}\times\dfrac{3}{4}$

3. Round the following numbers to the indicated places.

(2) a. 4.0726 to the nearest thousandth is **4.073**.

(2) b. 162.5 to the nearest ten is **160**.

(2) c. 0.5734 to the nearest tenth is **0.6**.

(2) d. 4.753 24 to the nearest ten thousandth is **4.7532**.

(2) e. 28.953 to the nearest tenth is **29.0**.

(2) f. 0.506 to the nearest hundredth is **0.51**.

Note: Answer e **must** have the zero in the tenths place to show that it has been rounded to tenths. 29 is **not** an acceptable answer.

4. Express the following as decimals and indicate if they are terminating or repeating decimals.

(2) a. $\frac{7}{15} = 0.4\overline{6}$ (repeating)

(2) b. $\frac{5}{8} = 0.625$ (terminating)

5. Do the following conversions.

(2) a. Convert $10\frac{7}{8}$ to a decimal and then to a percent.

$$10\frac{7}{8} = 10.875$$
$$= 1087.5\%$$

(2) b. Convert 78.3% to a decimal and then to a fraction.

$$78.3\% = 0.783$$
$$= \frac{783}{1000}$$

(2) c. Convert 3.75 to a percent and then to a fraction.

$$3.75 = 375\%$$
$$= \frac{375}{100}$$
$$= \frac{15}{4}$$

(4) 6. Arrange the following numbers in descending order.

$3\frac{1}{3}$, $\frac{30}{8}$, $-6\frac{1}{4}$, 3.7, 35%

Write each as a decimal.

$3.\overline{3}$, 3.75, −6.25, 3.7, 0.35

In descending order, the decimals are be listed as follows:

3.75, 3.7, $3.\overline{3}$, 0.35, −6.25

The numbers in descending order are as follows:

$\frac{30}{8}$, 3.7, $3\frac{1}{3}$, 35%, $-6\frac{1}{4}$

(4) 7. Circle the numbers that are **not** integers.

A. $\frac{14}{7}=2$ (B.) 6.23

(C.) $\pi$ D. −9

(E.) $2\frac{1}{4}$ F. 0

(G.) $\frac{-3}{5}$

B, C, E, and G are not integers.

**Topic 1**

__42__ marks

## Topic 2: Irrational Number System

1. Find the square root of the following:

(2) a. $\sqrt{-121} = \sqrt{121(-1)}$
$= \sqrt{121} \times \sqrt{-1}$
$= 11i$

(2) b. $\sqrt{-\frac{4}{9}} = \sqrt{\frac{4}{9}(-1)}$
$= \sqrt{\frac{4}{9}} \times \sqrt{-1}$
$= \frac{2}{3}i$

(2) c. $\sqrt{-0.38} = \sqrt{0.38(-1)}$
$= \sqrt{0.38} \times \sqrt{-1}$
$\doteq 0.62i$

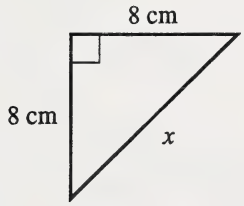
(6) 2. Show that the product of $5.\overline{36}$ and $0.\overline{7}$ is a repeating decimal.
(Hint: Note that $5.\overline{36} = 5\frac{4}{11}$ and $0.\overline{7} = \frac{7}{9}$.)

$5\frac{4}{11} \times \frac{7}{9} = \frac{59}{11} \times \frac{7}{9}$
$= \frac{413}{99}$
$= 4.\overline{17}$

(3) 3. Change $i\sqrt{15}$ to a form such that you have the square root of a negative number.

$$i\sqrt{15} = \sqrt{-1} \times \sqrt{15}$$
$$= \sqrt{-15}$$

(6) 4. Two sides of a right triangular bracket are each 8 cm in length. How long is the third side? Round your answer to the nearest hundredth.




$$x^2 = 8^2 + 8^2$$
$$= 64 + 64$$
$$= 128$$
$$x = \sqrt{128}$$
$$\doteq 11.31 \text{ cm}$$

(3) 5. Circle the numbers that are irrational.

A. $2 - 3i$

B. $12.1\overline{7}$

C. $\sqrt{\frac{9}{16}}$

(D.) $\sqrt{5}$

(E.) $\sqrt{\frac{4}{7}}$

D and E are irrational.

(4) 6. What is the length of each side of a square whose area is equal to the area of a rectangle that measures 7 cm by 8 cm? Round your answer to the nearest tenth.

$$\text{Area of rectangle} = l \times w$$
$$= 7 \times 8$$
$$= 56 \text{ cm}^2$$

$$\text{Area of square} = s^2$$
$$56 = s^2$$
$$s = \sqrt{56}$$
$$\doteq 7.5 \text{ cm}$$

(4) 7. If the area of a circle is 24 $\text{cm}^2$, what is the length of its radius? Round your answer to the nearest hundredth. (Hint: Note that $A = \pi r^2$ and $\pi \doteq 3.14$.)

$$A = \pi r^2$$
$$24 = 3.14 r^2$$
$$r^2 = \frac{24}{3.14}$$
$$r = \sqrt{\frac{24}{3.14}}$$
$$\doteq 2.76 \text{ cm}$$

**Topic 2**

__32__ marks

## Topic 3: Relationships Among Number Systems

1. Prize money of \$56 is shared equally among seven people. A simple way of dividing this sum equally is by using a fraction $\left(\frac{1}{7} \text{ of } \$56\right)$.

(3) a. Explain why you would not use the decimal or percent value of $\frac{1}{7}$ to find the equal shares of this money.

The decimal equivalent for $\frac{1}{7}$ is repeating. The percent will be an approximate value. You would not get an exact amount using the decimal or the percent.

(3) b. What is each person's share?

$$\frac{1}{7} \times 56 = \frac{56}{7}$$
$$= 8$$

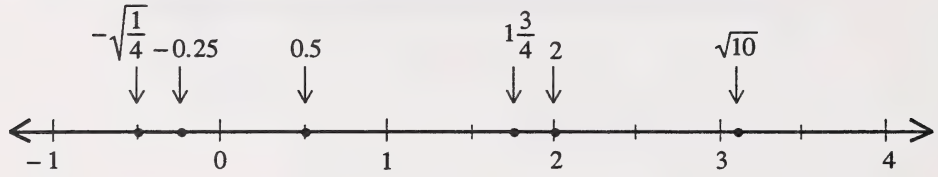
Each person gets \$8.

(6) 2. Locate the following numbers on the number line given.

$2, 0.5, \sqrt{10}, 1\frac{3}{4}, -0.25, -\sqrt{\frac{1}{4}}$

Write each number in decimal form.

$2, 0.5, \sqrt{10} \doteq 3.16, 1\frac{3}{4} = 1.75, -0.25, -\sqrt{\frac{1}{4}} = -\frac{1}{2} = -0.5$




(4) 3. Arrange in ascending order.

$\sqrt{6}, 6, \sqrt{17}, \sqrt{9}, 2, \sqrt{15}, \frac{3}{4}$

Write each as decimals.

2.45, 6, 4.12, 3, 2, 3.87, 0.75

Ascending order of these decimal numbers is as follows:

0.75, 2, 2.45, 3, 3.87, 4.12, 6

Ascending order of the given numbers is as follows:

$\frac{3}{4}, 2, \sqrt{6}, \sqrt{9}, \sqrt{15}, \sqrt{17}, 6$

4. A shopkeeper marked an item **Reduced by 70%**.

(1) a. Why is a percent used instead of a decimal, or a fraction?

A percent is used for ease of comparison. Seventy percent shows a lot of saving. Decimals and fractions are more difficult for calculation.

(3) b. If the regular price of the item was $26, what is its reduced price?

$$26 \times \frac{70}{100} = 18.2$$
$$26 - 18.2 = 7.8$$

The reduced price is $7.80.

(3) c. If the regular price of the item was $78.24, what is the amount of the discount? Round your answer to the nearest cent.

$$78.24 \times \frac{70}{100} = 54.768$$

The discount is $54.77.

(3) d. If the price of the item is reduced by $14.00, what was the original price?

Since $14 is the reduction, it is equal to 70% of the original price.

$$\frac{14}{x} = \frac{70}{100}$$
$$70x = 1400$$
$$x \doteq 20$$

The original price was $20.00.

**Topic 3**

__26__ marks

# MATHEMATICS 10 LEARNING

## FACILITATOR'S MANUAL

## UNIT 2

# OPERATIONS ON POLYNOMIALS

Distance Learning

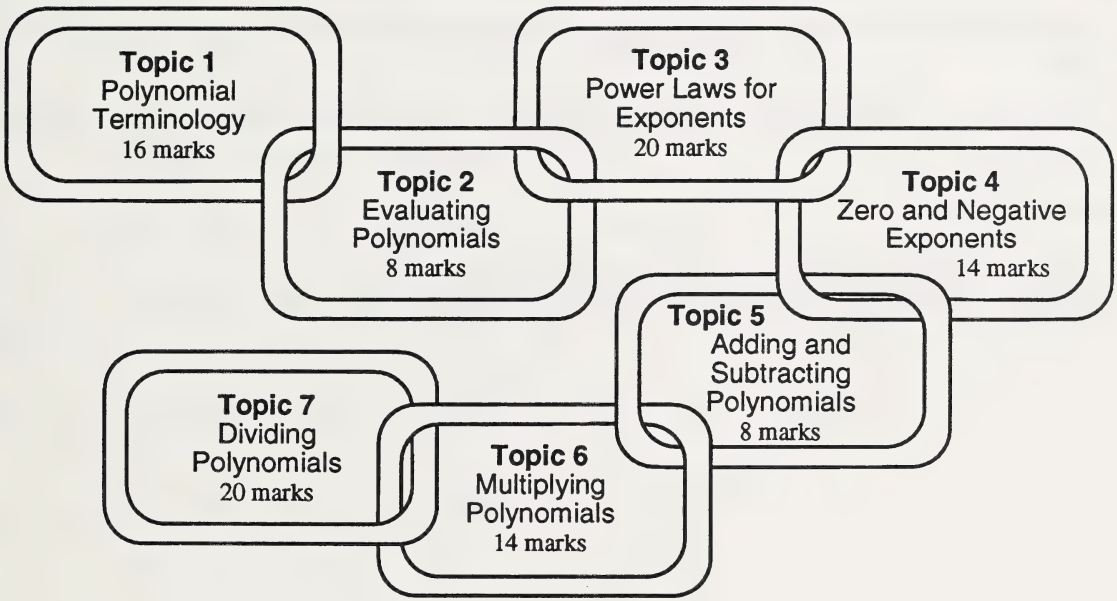
Alberta EDUCATION

## Unit 2 Overview

This unit called Operations on Polynomials is worth 14% of the course mark.

The unit consists of seven topics. Each topic has an assignment in the Assignment Booklet.

### Unit 2
### Operations on Polynomials



## Concept Introduction Possibilities

- Use a dictionary to define terms.
- Substitute numerical values to evaluate polynomials.
- Use power laws to multiply and divide terms.
- Show how to combine like terms.
- Distinguish between expanding and factoring.
- Use algebra tiles and binomial grids to find the expanded form of binomials.
- Compare algebraic factoring and factoring using a binomial grid.

## Possible Presentation Activities

- Using algebra tiles and binomial grids, show how to find the area of a rectangle when given a length and a width.

- Using algebra tiles and binomial grids, show how to find an unknown length of a rectangle when given the area and the width.

## Marking Guidelines

- If the answer is correct, give full marks whether or not work is shown.
- If the answer is incorrect and work is shown, give partial marks for **correct** work.

## Topic 1: Polynomial Terminology

1. Name the numerical coefficient in each term.

(1) a. $(-x)^2$ **1** $(-x)^2 = 1x^2$

(1) b. $\left(-3x^2\right)^3$ **−27** $\left(-3x^2\right)^3 = (-3)^3 x^{2\times 3} = -27x^6$

(1) c. $6(2x)^2$ **24** $6(2x)^2 = 6\left(4x^2\right) = 24x^2$

(1) d. $-(3x)^2$ **−9** $-(3x)^2 = -\left(9x^2\right) = -9x^2$

2. Determine the degree of each term.

(1) a. $2x^2y^3$ **5th degree** $(2+3)$

(1) b. $\frac{x^3}{x}$ **2nd degree** $(3-1)$

(1) c. $\frac{1}{3}(6x)$ **1st degree**

(1) d. $-2\left(x^2y\right)^2$ **6th degree** $-2\left(x^2y\right)^2 = -2x^4y^2$

3. Classify the following polynomials. Determine the number of terms, the degree, and the number of variables in each.

(3) a. $12s^3t^2v$

Number of terms = **monomial** (monomial, binomial, trinomial)

degree = **6** ((1, 2, 3, ...)

Number of variables = **3** (1, 2, 3, ...)

(3) b. $6x^3y^3 - 3x^2y + \frac{1}{2}xy^2$

Number of terms = **trinomial** (monomial, binomial, trinomial)

degree = **6** (1, 2, 3, ...)

Number of variables = **2** (1, 2, 3, ...)

(2) 4. In the expression $x^n$, the exponent is ***n*** and the base is ***x***.

**Topic 1**

16 marks

## Topic 2: Evaluating Polynomials

1. Given $a=-1$, $b=2$, and $c=6$, evaluate the following.

(2) a. $14-abc^2 = 14-(-1)(2)(6)^2$
$= 14-(-2)(36)$
$= 14-(-72)$
$= 86$

(2) b. $2a^2+3ab-4b^2 = 2(-1)^2+3(-1)(2)-4(2)^2$
$= 2(1)+3(-2)-4(4)$
$= 2-6-16$
$= -20$

2. If $f(y)=5y^2-y+7$, find the following values.

(2) a. $f(3)=5(3)^2-(3)+7$
$= 5(9)-3+7$
$= 45-3+7$
$= 49$

(2) b. $f(-2)=5(-2)^2-(-2)+7$
$= 5(4)+2+7$
$= 20+2+7$
$= 29$

**Topic 2**

__8__ marks

## Topic 3: Power Laws and Exponents

1. Simplify the following.

(2) a. $y^2 \times y^{12} = y^{2+12}$
$= y^{14}$

(2) b. $\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2}$
$= \frac{16}{25}$

(2) c. $e^7 \times 3e = 3e^{7+1}$
$= 3e^8$

(2) d. $6b^4 \div 2b = \frac{6}{2} \times \frac{b^4}{b^1}$
$= 3b^{4-1}$
$= 3b^3$

2. Complete the following sentences.

(1) a. In the statement $2^4 = 16$, the 2 is called the **base**.

(1) b. According to the quotient law for exponents, $x^a \div x^b =$ $x^{a-b}$.

3. Simplify the following.

(2) a. $\left(c^4\right)^8 = c^{4\times 8}$

$= c^{32}$

(2) b. $\left(x^2 b^3\right)^7 = x^{2\times 7} b^{3\times 7}$

$= x^{14} b^{21}$

(2) c. $\left(y^2\right)^2 \times y^{12} = y^{2\times 2} \times y^{12}$

$= y^4 y^{12}$

$= y^{16}$

(2) d. $\left(b^2 c^3\right)^4 = b^{2\times 4} c^{3\times 4}$

$= b^8 c^{12}$

(2) e. $\left(\frac{c^2}{b}\right)^2 \times b = \frac{c^{2\times 2}}{b^2} \times b^1$

$= c^4 b^{1-2}$

$= c^4 b^{-1}$ or $\frac{c^4}{b}$

**Topic 3**

__20__ marks

## Topic 4: Zero and Negative Exponents

1. Simplify the following.

(2) a. $4^{-3} = \frac{1}{4^3}$

$= \frac{1}{64}$

(2) b. $\left(x^{-1}y^2\right)^{-3} = \left(x^{-1}\right)^{-3}\left(y^2\right)^{-3}$

$= x^3 y^{-6}$ or $\frac{x^3}{y^6}$

(2) c. $c^0 \times c^8 = c^{0+8}$

$= c^8$

(2) d. $\left(\frac{-3}{4}\right)^{-3} = \left(\frac{-4}{3}\right)^3$

$= \frac{-64}{27}$

(2) e. $\left(\frac{-2}{c}\right)^{-2} = \left(\frac{-c}{2}\right)^2$

$= \frac{c^2}{4}$

(2) f. $x^0 + 3(x+y)^0 = 1 + 3(1)$

$= 1 + 3$

$= 4$

(2) g. $\left[\left(\frac{w^2 + x^3}{z^4}\right)^2\right]^0 = 1$

**Topic 4**

__14__ marks

## Topic 5: Adding and Subtracting Polynomials

1. Simplify each of the following.

(2) a. $12a^3+4a^2-a-2a^3+7a^2-7a=12a^3-2a^3+4a^2+7a^2-a-7a$
$=10a^3+11a^2-8a$

(2) b. $(2n^2-5)+(4n^2+8n)=2n^2+4n^2-5+8n$
$=6n^2+8n-5$

(2) c. $(p^2+3pm-2m^2)-(-5p^2-8pm+m^2)=p^2+3pm-2m^2+5p^2+8pm-m^2$
$=p^2+5p^2+3pm+8pm-2m^2-m^2$
$=6p^2+11pm-3m^2$

(2) 2. Simplify $(c^2e+2ch-h^3)+(h^3+4ch-2c^2e)$; then evaluate if $c=-1$, $e=2$ and $h=-3$.

$(c^2e+2ch-h^3)+(h^3+4ch-2c^2e)=c^2e+2ch-h^3+h^3-4ch-2c^2e$
$=c^2e-2c^2e+2ch-4ch-h^3+h^3$
$=-c^2e-2ch$

$-c^2e-2ch=-(-1)^2(2)-2(-1)(-3)$
$=-(2)-2(3)$
$=-2-6$
$=-8$

**Topic 5**

8 marks

## Topic 6: Multiplying Polynomials

(2) 1. Multiply $(2x+3)$ by $(x-3)$.

$$(2x+3)(x-3) = 2x^2 - 6x + 3x - 9$$
$$= 2x^2 - 3x - 9$$

2. Expand the following.

(2) a. $-2x(ax^3 - bx^2 + c) = -2x(ax^3) - 2x(-bx^2) - 2x(c)$
$$= -2ax^4 + 2bx^3 - 2cx$$

(2) b. $(3x-2)(4x-5) = 12x^2 - 15x - 8x + 10$
$$= 12x - 23x + 10$$

(2) c. $(a+b)(a-b) = a^2 - ab + ab - b^2$
$$= a^2 - b^2$$

(2) d. $(1-m^4)(1+m^4) = 1 + m^4 - m^4 - m^8$
$$= 1 - m^8$$

3. Determine the following.

(2) a. Find the square of $2x+7y$.

$$\begin{aligned}(2x+7y)^2 &= (2x+7y)(2x+7y)\\ &= 4x^2+14xy+14xy+49y^2\\ &= 4x^2+28xy+49y^2\end{aligned}$$

(2) b. $$\begin{aligned}(x-3)^2 &= (x-3)(x-3)\\ &= x^2-3x-3x+9\\ &= x^2-6x+9\end{aligned}$$

**Topic 6**

14 marks

## Topic 7: Dividing Polynomials

1. Simplify the following.

(2) a. $\dfrac{6x^4y^2}{3x^2y^2} = \dfrac{6}{3}x^{4-2}y^{2-2}$

$= 2x^2y^0$

$= 2x^2$

(2) b. $\dfrac{20x^{12}}{5x^4} = \dfrac{20}{5}x^{12-4}$

$= 4x^8$

2. Divide the following.

(2) a. $\dfrac{5x^5 - 10x^4 + 5x^3}{-5x^3} = \dfrac{5x^5}{-5x^3} - \dfrac{10x^4}{-5x^3} + \dfrac{5x^3}{-5x^3}$

$= -1x^{5-3} + 2x^{4-3} - 1x^{3-3}$

$= -x^2 + 2x - 1$

(2)

b. $\dfrac{8x^2y^2-16xy^3}{4xy}=\dfrac{8x^2y}{4xy}-\dfrac{16xy^3}{4xy}$

$=2x^{2-1}y^{2-1}-4x^{1-1}y^{3-1}$

$=2xy-4x^0y^2$

$=2xy-4y^2$

3. Use long division to divide the following. State the quotient and the remainder.

(4)

a. $\dfrac{9t^2+9t-45}{3t-5}$

$$3t-5\overline{)9t^2+9t-45}$$

$3t+8$

$9t^2-15t$

$24t-45$

$24t-40$

$-5$

The quotient is $3t+8$.
The remainder is $-5$.

④ b. $\dfrac{x^3+6x^2-x-30}{x-2}$

$$
\begin{array}{r}
x^2+8x+15 \\
x-2\overline{)\,x^3+6x^2-x-30} \\
\underline{x^3-2x^2} \\
8x^2-x \\
\underline{8x^2-16x} \\
15x-30 \\
\underline{15x-30} \\
0
\end{array}
$$

The quotient is $x^2+8x+15$.
The remainder is 0.

④ c. $\dfrac{x^2-2}{x+2}$

$$
\begin{array}{r}
x-2 \\
x+2\overline{)\,x^2+0x-2} \\
\underline{x^2+2x} \\
-2x-2 \\
\underline{-2x-4} \\
+2
\end{array}
$$

The quotient is $x-2$.
The remainder is 2.

**Topic 7**

__20__ marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL

## UNIT 3

## EQUATIONS AND INEQUALITIES

Distance Learning

Alberta EDUCATION

## Unit 3 Overview

This unit called Equations and Inequalities is worth 10% of the course mark.

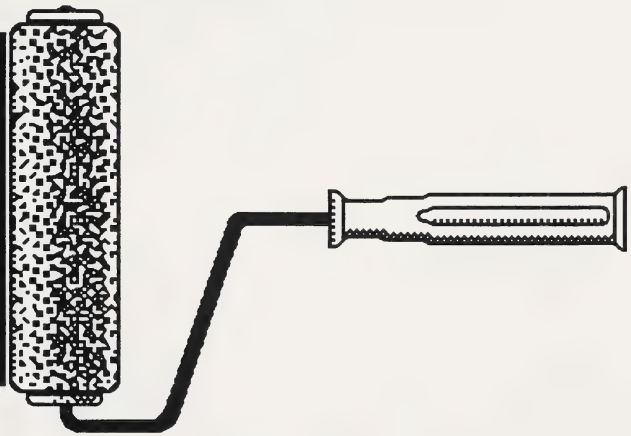
The unit consists of three topics. Each topic has an assignment in the Assignment Booklet.

### Unit 3 Equations and Inequalities

**Topic 1** Solving and Verifying Linear Equations
35 marks

**Topic 2** Inequalities
30 marks

**Topic 3** Simple Quadratic and Radical Equations
35 marks



### Concept Introduction Possibilities

Work through several examples pointing out the skills required in order to solve equations.

- Emphasize that whatever is done to one side of an equation must also be done to the other side to maintain equivalence. For example, $2x-12=48$.
- Show how to collect like terms. For example, $3x-2=x+6$.
- Demonstrate the use of the distributive property. For example, $5x-2(x+3)=9$.
- Simplify by multiplying by the lowest common denominator (L.C.D). For example, $\frac{3x-1}{2}-\frac{3x-2}{4}=5$.

### Marking Guidelines

- If the answer is correct, give full marks whether or not work is shown.
- If the answer is incorrect and work is shown, give partial marks for correct work.

## Topic 1: Solving and Verifying Linear Equations

1. Solve and verify the following.

(2)

a. $-13 = 6x - 25$

$-13 + 25 = 6x - 25 + 25$

$12 = 6x$

$\frac{\overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}}} = \frac{\overset{1}{\cancel{6}}x}{\underset{1}{\cancel{6}}}$

$2 = x$

$x = 2$

Verify:

| LS | RS |
|---|---|
| $-13$ | $6x - 25$ |
| $-13$ | $6(2) - 25$ |
| $-13$ | $12 - 25$ |
| $-13$ | $-13$ |

LS = RS

(2)

b. $3 - 4(x+2) = 5(x-4) - 3$

$3 - 4x - 8 = 5x - 20 - 3$

$-4x - 5 = 5x - 23$

$-4x - 5 - 5x = 5x - 23 - 5x$

$-9x - 5 = -23$

$-9x - 5 - 5 = -23 + 5$

$-9x = -18$

$\frac{\overset{1}{\cancel{-9}}x}{\underset{1}{\cancel{-9}}} = \frac{\overset{1}{\cancel{-18}}}{\underset{1}{\cancel{-9}}}$

$x = 2$

Verify:

| LS | RS |
|---|---|
| $3 - 4(x+2)$ | $5(x-4) - 3$ |
| $3 - 4[(2)+2]$ | $5[(2)-4] - 3$ |
| $3 - 4(4)$ | $5(-2) - 3$ |
| $3 - 16$ | $-10 - 3$ |
| $-13$ | $-13$ |

LS = RS

④

c. $(x+1)(x-2)=x(x+3)-10$

$x^2-2x+x-2=x^2+3x-10$

$x^2-x-2=x^2+3x-10$

$x^2-x^2-x-2=x^2-x^2+3x-10$

$-x-2=3x-10$

$-x-3x-2+2=3x-3x-10+2$

$-4x=-8$

$x=2$

Verify:

| LS | RS |
|---|---|
| $(x+1)(x-2)$ | $x(x+3)-10$ |
| $(2+1)(2-2)$ | $2(2+3)-10$ |
| $3(0)$ | $2(5)-10$ |
| $0$ | $10-10$ |
| $0$ | $0$ |

LS = RS

④

d. $(9x-3)(x-1)-8=(3x-1)^2$

$9x^2-3x-9x+3-8=9x^2-3x-3x+1$

$9x^2-12x-5=9x^2-6x+1$

$9x^2-9x^2-12x-5=9x^2-9x^2-6x+1$

$-12x-5=-6x+1$

$-12x+6x-5+5=-6x-6x+1+5$

$-6x=6$

$x=-1$

Verify:

| LS | RS |
|---|---|
| $(9x-3)(x-1)-8$ | $(3x-1)^2$ |
| $[9(-1)-3](-1-1)-8$ | $[3(-1)-1]^2$ |
| $(-9-3)(-2)-8$ | $(-3-1)^2$ |
| $(-12)(-2)-8$ | $(-4)^2$ |
| $24-8$ | $16$ |
| $16$ | $16$ |

LS = RS

(4) 2. Solve and verify $0.25(3x-1)=2.375$.

$$0.25(3x-1)=2.375$$
$$0.75x-0.25=2.375$$
$$0.75x-0.25+0.25=2.375+0.25$$
$$0.75x=2.625$$
$$x=3.5$$

Verify:

| LS | RS |
|---|---|
| $0.25(3x-1)$ | 2.375 |
| $0.25[3(3.5)-1]$ | 2.375 |
| $0.25(9.5)$ | 2.375 |
| 2.375 | 2.375 |

LS = RS

3. Write an algebraic expression to represent each phrase.

(1) a. three more than triple a number

$3x+3$

(1) b. your age three years ago

$x-3$

(1) c. Twice the length decreased by eight is thirty-two.

$2l-8=32$

(1) d. The sum of two consecutive integers is 121.

$x+(x+1)=121$

(1) e. Five times Tim's age is 85.

$5x=85$

(3) 4. Solve $\frac{5x-4}{3}+5=\frac{3x+1}{4}$.

$$\frac{5x-4}{3}+5=\frac{3x+1}{4}$$

$$\cancel{(12)}^{4}\frac{5x-4}{\cancel{3}_{1}}+(12)5=\cancel{(12)}^{3}\frac{3x+1}{\cancel{4}_{1}}$$

$$4(5x-4)+60=3(3x+1)$$

$$20x-16+60=9x+3$$

$$20x+44=9x+3$$

$$20x-9x=3-44$$

$$11x=-41$$

$$x=-\frac{41}{11}$$

(5) 5. Three less than six times an integer is 123. Determine the integer.

Let $x$ be the integer.

$$6x-3=123$$

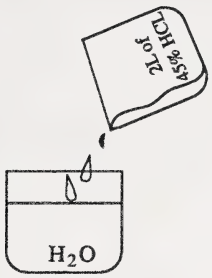
$$6x-3+3=123+3$$

$$6x=126$$

$$x=21$$

The integer is 21.

⑥ 6. How much water must be in the container to produce a 30% hydrochloric acid (HCL) solution when 2 L of a 45% hydrochloric acid solution is added.




Let $x$ represent the amount of water in the container. After the acid has been added to the water, there will be $2+x$ litres of 30% HCL solution, which contains $0.3(2+x)$ litres of acid. The volume of pure acid in two litres of 45% HCL solution is $(2\text{ L})(45\%)=(2\text{ L})(0.45)$.

The amount of acid in each solution remains the same.

$$0.45(2)=0.3(2+x)$$
$$0.9=0.6+0.3x$$
$$0.3=0.3x$$
$$x=1\text{ L}$$

There must be 1 L of water in the container.

**Topic 1**

__35__ marks

## Topic 2: Inequalities

1. Solve and graph. In each expression, $x \in R$

(4) a. $\frac{5x-3}{4} < -7$

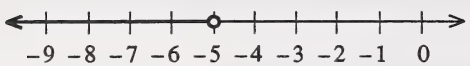
$(5x-3) < 4(-7)$ (Clear the denominator by multiplying each side by 4.)

$5x-3 < -28$

$5x < -25$

$x < \frac{-25}{5}$

$x < -5$

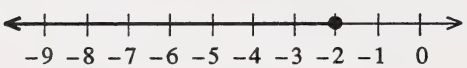



(4) b. $-7x+4 \geq 6-6x$

$-7x+6x \geq 6-4$

$-x \geq 2$

$x \leq -2$ (Divide both sides by $-1$, and reverse the inequality sign.)



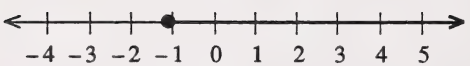

(4) c. $2(x+3) \geq 3-(x+1)$

$2x+6 \geq 3-x-1$

$2x+x \geq 3-1-6$

$3x \geq -4$

$x \geq \frac{-4}{3}$



(3) 2. The height of a triangle is 8 cm. What possible bases ($b$) would keep the area less than 48 $cm^2$?

$$\text{Area} = \frac{1}{2}bh$$

$$48 > \frac{1}{2}(b)(8)$$

$$48 > 4b$$

$$12 > b$$

$$b < 12$$

The base must be less than 12 cm long.

(3) 3. In fifteen games the Flames scored over fifty goals. How many goals did they average per game?

Let $a$ be the average goals per game.

$$\text{Average} = \frac{\text{total goals}}{\text{number of games}}$$

$$\text{Average} \times \text{number of games} = \text{total goals}$$

$$a(15) > 50$$

$$a > \frac{50}{15}$$

$$a > 3.34$$

The Flames averaged over three goals per game.

⑥ 4. Solve and verify $2(x-3)-4x>28$.

$2(x-3)-4x>28$ (Use the distributive property to remove the parentheses.)

$2x-6-4x>28$

$-2x-6>28$

$-2x>34$

$x<-17$ (Dividing by a negative number changes the direction of the inequality.)

Verify:

Let $x=-16$, which is a number **greater than** $-17$.

| LS | | RS |
|---|---|---|
| $2(x-3)-4x$ | | 28 |
| $2[(-16)-3]-4(-16)$ | | 28 |
| $2(-19)+64$ | | 28 |
| $-38+64$ | | 28 |
| 26 | | 28 |
| LS | $\not>$ | RS |

The inequality is not true for $x>-17$.

Let $x=-18$, which is a number **less than** $-17$.

| LS | | RS |
|---|---|---|
| $2(x-3)-4x$ | | 28 |
| $2[(-18)-3]-4(-18)$ | | 28 |
| $2(-21)+72$ | | 28 |
| $-42+72$ | | 28 |
| 30 | | 28 |
| LS | $>$ | RS |

Therefore, the inequality is true for $x<-17$.

These tests show that the inequality is true for $x<-17$.

⑥ 5. Solve, verify, and graph $(x+2)(3x-1)<x(3x+2)+1$.

$$
\begin{aligned}
(x+2)(3x-1) &< x(3x+2)+1 \\
3x^2-x+6x-2 &< 3x^2+2x+1 \\
3x^2+5x-2 &< 3x^2+2x+1 \\
3x^2-3x^2+5x-2 &< 3x^2-3x^2+2x+1 \\
5x-2 &< 2x+1 \\
5x-2x-2+2 &< 2x-2x+1+2 \\
3x &< 3 \\
x &< 1
\end{aligned}
$$

Verify:

Let $x=0$ which is less than 1.

| LS | | RS |
|---|---|---|
| $(x+2)(3x-1)$ | | $x(3x+2)+1$ |
| $(0+2)[3(0)-1]$ | | $0[3(0)+2]+1$ |
| $2(0-1)$ | | $0(0+2)+1$ |
| $2(-1)$ | | $0(2)+1$ |
| $-2$ | | $0+1$ |
| $-2$ | | $1$ |
| LS | $<$ | RS |

The inequality is true for $x<1$.

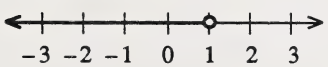
Let $x=2$ which is greater than 1.

| LS | | RS |
|---|---|---|
| $(x+2)(3x-1)$ | | $x(3x+2)+1$ |
| $(2+2)[3(2)-1]$ | | $2[3(2)+2]+1$ |
| $4(6-1)$ | | $2(6+2)+1$ |
| $4(5)$ | | $2(8)+1$ |
| $20$ | | $16+1$ |
| $20$ | | $17$ |
| LS | $\not<$ | RS |

The inequality is not true for $x>1$.

The tests show the inequality is true for $x<1$.

The graph is as follows:




**Topic 2**

30 marks

## Topic 3: Simple Quadratic and Radical Equations

1. Solve and verify the following.

④

a. $\sqrt{3x-5}=7$ (Square each side.)

$3x-5=49$

$3x=49+5$

$3x=54$

$x=18$

Verify:

| LS | RS |
|---|---|
| $\sqrt{3x-5}$ | 7 |
| $\sqrt{3(18)-5}$ | 7 |
| $\sqrt{54-5}$ | 7 |
| $\sqrt{49}$ | 7 |
| 7 | 7 |

LS = RS

④

b. $\sqrt{\frac{x-3}{2}}=6$ (Square each side.)

$\frac{x-3}{2}=36$

$x-3=72$

$x=75$

Verify:

| LS | RS |
|---|---|
| $\sqrt{\frac{x-3}{2}}$ | 6 |
| $\sqrt{\frac{75-3}{2}}$ | 6 |
| $\sqrt{\frac{72}{2}}$ | 6 |
| $\sqrt{36}$ | 6 |
| 6 | 6 |

LS = RS

④

2. The area of a bathroom wall is 20 000 $cm^2$. It is covered with 200 square tiles. Find the dimensions of one of the tiles.

Area of each tile $= x^2$

$200x^2 = 20\,000$

$x^2 = 100$

$x = 10$ cm

Each tile is 10 cm by 10 cm.

3. Solve and verify the following.

②

a. $\sqrt{x} = 9$

$x = 81$

Verify:

| LS | | RS |
|---|---|---|
| $\sqrt{x}$ | | 9 |
| $\sqrt{81}$ | | 9 |
| 9 | | 9 |
| LS | = | RS |

Therefore $x = 81$ is a solution.

④ b. $9x^2 - 64 = 0$

$$9x^2 = 64$$

$$x^2 = \frac{64}{9}$$

$$\pm\sqrt{x^2} = \pm\sqrt{\frac{64}{9}}$$

$$x = \pm\frac{8}{3}$$

Verify:

When $x = \frac{8}{3}$:

| LS | | RS |
|---|---|---|
| $9x^2 - 64$ | | 0 |
| $9\left(\frac{8}{3}\right)^2 - 64$ | | 0 |
| $9\left(\frac{64}{9}\right) - 64$ | | 0 |
| $64 - 64$ | | 0 |
| 0 | | 0 |
| LS | = | RS |

When $x = -\frac{8}{3}$:

| LS | | RS |
|---|---|---|
| $9x^2 - 64$ | | 0 |
| $9\left(-\frac{8}{3}\right)^2 - 64$ | | 0 |
| $9\left(\frac{64}{9}\right) - 64$ | | 0 |
| $64 - 64$ | | 0 |
| 0 | | 0 |
| LS | = | RS |

Therefore, $x = \frac{8}{3}$ and $x = -\frac{8}{3}$ are solutions.

③ c. $\sqrt{x-3} + 7 = 6$

$\sqrt{x-3} = -1$ (Square each side.)

$x - 3 = 1$

$x = 4$

Verify:

| LS | | RS |
|---|---|---|
| $\sqrt{x-3} + 7$ | | 6 |
| $\sqrt{4-3} + 7$ | | 6 |
| $1 + 7$ | | 6 |
| 8 | | 6 |
| LS | ≠ | RS |

Therefore, $x = 4$ is not a solution and it is called an extraneous root.

4. Solve and verify the following.

(4)

a. $3x^2 = 192$

$x^2 = \frac{192}{3}$

$x^2 = 64$

$x = \pm\sqrt{64}$

$= \pm 8$

Verify:

When $x = 8$:

| LS | | RS |
|---|---|---|
| $3x^2$ | | 192 |
| $3(8)^2$ | | 192 |
| $3 \times 64$ | | 192 |
| 192 | | 192 |
| LS | = | RS |

When $x = -8$:

| LS | | RS |
|---|---|---|
| $3x^2$ | | 192 |
| $3(-8)^2$ | | 192 |
| $3 \times 64$ | | 192 |
| 192 | | 192 |
| LS | = | RS |

Therefore, the solutions are $x = 8$ and $x = -8$.

⑥ b. $(3x-4)^2 = 196$

$3x-4 = \pm\sqrt{196}$

$3x-4 = \pm 14$

Case 1:

$3x-4=14$

$3x=18$

$x=6$

Verify:

| LS | RS |
|---|---|
| $(3x-4)^2$ | 196 |
| $[3(6)-4]^2$ | 196 |
| $(18-4)^2$ | 196 |
| $(14)^2$ | 196 |
| 196 | 196 |

LS = RS

Case 2:

$3x-4=-14$

$3x=-10$

$x=\frac{-10}{3}$

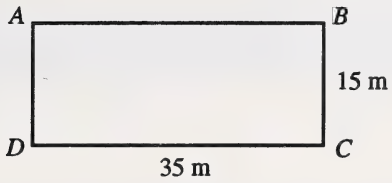
Verify:

| LS | RS |
|---|---|
| $(3x-4)^2$ | 196 |
| $\left[3\left(\frac{-10}{3}\right)-4\right]^2$ | 196 |
| $(-10-4)^2$ | 196 |
| $(-14)^2$ | 196 |
| 196 | 196 |

LS = RS

Therefore, 6 and $\frac{-10}{3}$ are the two solutions.

④ 5. To find the diagonal distance across a rectangle, you can use the formula $d = \sqrt{l^2 + w^2}$, where $l$ is the length and $w$ is the width. Use this formula to find the distance from $A$ to $C$ in the rectangle shown. Round your answer to the nearest metre.




$$d = \sqrt{l^2 + w^2}$$
$$= \sqrt{35^2 + 15^2}$$
$$= \sqrt{1225 + 225}$$
$$= \sqrt{1450}$$
$$= 38.078\,865\,53$$
$$\doteq 38$$

Therefore, the distance from $A$ to $C$ is about 38 m.

**Topic 3**

35 marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL

UNIT 4

## FACTORING POLYNOMIALS

Distance Learning

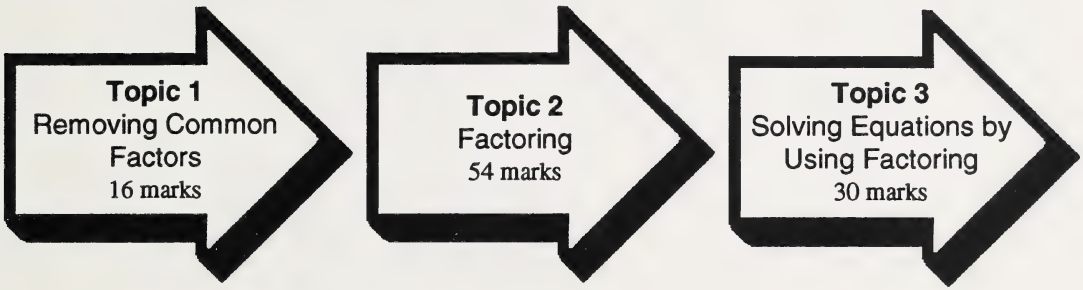
Alberta EDUCATION

## Unit 4 Overview

This unit called Factoring Polynomials is worth 13% of the course mark.

The unit consists of three topics. Each topic has an assignment in the Assignment Booklet.

**Unit 4**
**Factoring Polynomials**




### Concept Introduction Possibilities

- Review factoring integers using factor trees so that students will have a good grasp of the meaning of factors.
- Use algebra tiles and binomial grids to review multiplication of binomials.

### Possible Presentation Activities

- use of binomial grids
- use of algebra tiles

### Marking Guidelines

- If the answer is correct, give full marks whether or not work is shown.
- If the answer is incorrect and work is shown, give partial marks for **correct** work.

## Topic 1: Removing Common Factors

1. Factor each of the following completely.

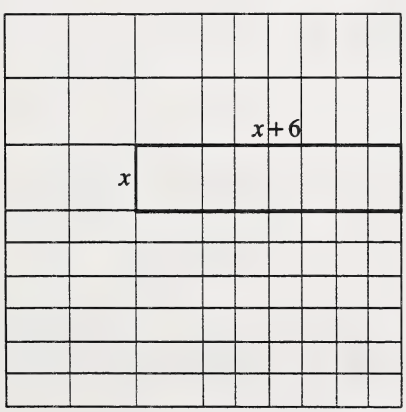
(2) a. $51a^3 - 34a^2b + 17ab^3 = 17a\left(3a^2 - 2ab + b^3\right)$

(2) b. $14x^2 + 7x + 21xy = 7x(2x + 1 + 3y)$
$= 7x(2x + 3y + 1)$

(2) c. $(x+2)^3 + (x+2)^2 = (x+2)^2[(x+2)+1]$
$= (x+2)^2(x+3)$

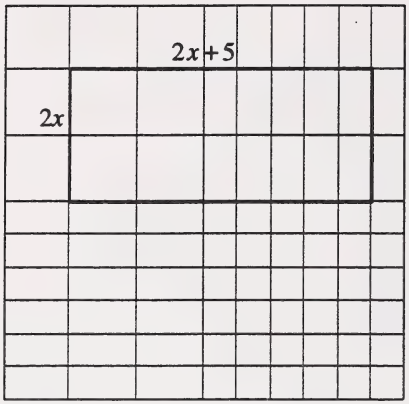
2. Factor each using a binomial grid.

(2) a. $x^2 + 6x$




$\therefore x^2 + 6x = x(x+6)$

(2) b. $4x^2 + 10x$




$\therefore 4x^2 + 10x = 2x(2x+5)$

3. Factor each completely.

(2) a. $ab + ce + ae + cb = ab + ae + cb + ce$
$= a(b+e) + c(b+e)$
$= (a+c)(b+e)$

(2) b. $6y^3 - 9y^2 + 10y - 15 = 3y^2(2y-3) + 5(2y-3)$
$= (3y^2 + 5)(2y-3)$

(2) 4. Explain why $3y$ is a factor of $9xy^2 - 18y$.

$3y$ is a factor of $9xy^2 - 18y$ because $3y$ is the largest factor that divides evenly into both $9x^2y$ and $-18y$.

OR

$3y$ is a factor of $9xy^2 - 18y$ because the product of $3y$ and $3xy - 6$ is equal to $9xy^2 - 18y$.

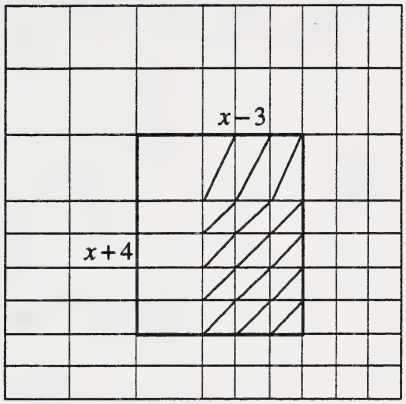
**Topic 1**

16 marks

## Topic 2: Factoring

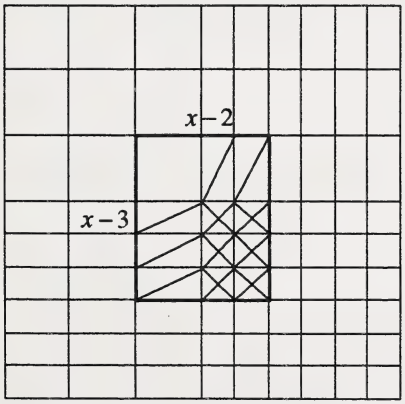
1. Factor each trinomial completely using a binomial grid.

(3) a. $x^2 + x - 12$



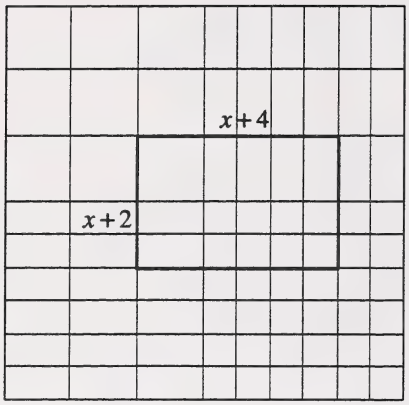

$\therefore x^2 + x - 12 = (x+4)(x-3)$

(3) b. $x^2 - 5x + 6$




$\therefore x^2 - 5x + 6 = (x-3)(x-2)$

(3) c. $x^2+6x+8$




$\therefore x^2+6x+8=(x+2)(x+4)$

2. Factor each of the following polynomials completely.

(3) a. $13x^2-26x-39=13\left(x^2-2x-3\right)$
$=13(x-3)(x+1)$

(2) b. $m^2-9m+18=(m-6)(m-3)$

(2) c. $18x^2+9x-2=(6x-1)(3x+2)$

(3) d. $5a^2-5a-150=5\left(a^2-a-30\right)$
$=5(a-6)(a+5)$

③ e. $144a^2+264ab+121b^2=(12a+11b)(12a+11b)$
$=(12a+11b)^2$

③ f. $3a^2+16ab+5b^2=(3a+b)(a+5b)$

③ g. $2a^3b+10a^2b-48ab=2ab\left(a^2+5a-24\right)$
$=2ab(a+8)(a-3)$

③ h. $36x^2-60xy+25y^2=(6x-5y)(6x-5y)$
$=(6x-5y)^2$

③ i. $16x^2-49y^2=(4x+7y)(4x-7y)$

③ j. $(x-y)^2-z^2=[(x-y)-z][(x-y)+z]$
$=(x-y-z)(x-y+z)$

③ k. $(2m+n)^2-m^2=(2m+n-m)(2m+n+m)$
$=(m+n)(3m+n)$

(3) l. $55c^3d - 26cd^3 = cd\left(55c^2 - 26d^2\right)$

$= cd\left(\sqrt{55}c + \sqrt{26}d\right)\left(\sqrt{55}c - \sqrt{26}d\right)$

(3) m. $(a+b)^3 - 64(a+b) = (a+b)\left[(a+b)^2 - 64\right]$

$= (a+b)(a+b-8)(a+b+8)$

3. Answer the questions using the following list.

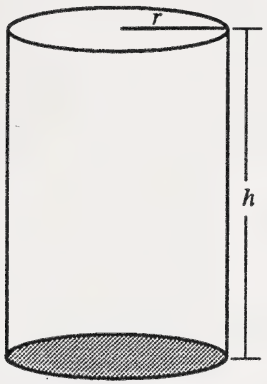
- $z^3 - y^3$
- $x^2 - 16$
- $x^2 + 16x + 64$
- $z^2 + y^2$
- $2x^2 - 4x + 9$

(1) a. Which one is a perfect square trinomial? **$x^2 + 16x + 64$**

(1) b. Which one is a difference of squares? **$x^2 - 16$**

(1) c. Which one is a product of conjugate binomials? **$x^2 - 16$**

4. The surface area of a cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$, where $r$ is the radius of the base and $h$ is the height of the cylinder.




(2) a. Rewrite the formula in factored form.

$$SA = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r(r + h)$$

(3) b. If the radius of its base is 8 cm and its height is 12 cm, what is the surface area of the cylinder? $(\pi = 3.14)$

$SA = 2\pi r(r + h)$

$= 2(3.14)(8)(8 + 12)$

$= 6.28(8)(20)$

$= 1004.8 \text{ cm}^2$

or $SA = 2\pi r^2 + 2\pi rh$

$= 2(3.14)(8)^2 + 2(3.14)(8)(12)$

$= 401.92 + 602.88$

$= 1004.8 \text{ cm}^2$

**Topic 2**

54 marks

## Topic 3: Solving Equations by Using Factoring

1. Find the zeros for each of the following.

③ a. $8x^2+80x+192=0$

$8\left(x^2+10x+24\right)=0$

$8(x+4)(x+6)=0$

$x+4=0$ or $x+6=0$

$x=-4$ $\quad$ $x=-6$

The real zeros are $x=-4$ and $x=-6$.

③ b. $-8x^2+48x-72=0$

$-8\left(x^2-6x+9\right)=0$

$-8(x-3)(x-3)=0$

$-8(x-3)^2=0$

$x-3=0$

$x=3$

The real zero is $x=3$.

2. Solve each quadratic equation by factoring.

② a. $x^2+5x-6=0$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x+6=0$ or $x-1=0$

$x=-6$ $\quad$ $x=1$

Therefore, $x=-6$ and $x=1$.

(2) b. $2x^2+2=5x$

$$2x^2+2=5x$$
$$2x^2-5x+2=0$$
$$(2x-1)(x-2)=0$$

$2x-1=0$ or $x-2=0$

$2x=1$ $\qquad$ $x=2$

$x=\frac{1}{2}$

Therefore, $x=\frac{1}{2}$ and $x=2$.

(2) c. $x^2+6x+8=0$

$$x^2+6x+8=0$$
$$(x+2)(x+4)=0$$

$x+2=0$ or $x+4=0$

$x=-2$ $\qquad$ $x=-4$

Therefore, $x=-2$ and $x=-4$.

(2) d. $x^3=16x$

$$x^3=16x$$
$$x^3-16x=0$$
$$x\left(x^2-16\right)=0$$
$$x(x-4)(x+4)=0$$

$x=0$ or $x-4=0$ or $x+4=0$

$x=4$ $\qquad$ $x=-4$

Therefore, $x=0$, $x=4$, and $x=-4$.

④ 3. Two numbers differ by 6. The sum of the squares of the numbers is 218. Find the numbers.

Let $x$ be the larger number.
Let $y$ be the smaller number.

$x-y=6$

$x=y+6$

$x^2+y^2=218$

$(y+6)^2+y^2=218$

$y^2+12y+36+y^2=218$

$2y^2+12y+36=218$

$2y^2+12y-182=0$

$2(y^2+6y-91)=0$

$2(y+13)(y-7)=0$

$y+13=0$ or $y-7=0$

$y=-13$ $y=7$

If $y=7$, $x=y+6$

$=13$

If $y=-13$, $x=y+6$

$=-7$

OR

$x-y=6$

$-y=-x+6$

$y=x-6$

$x^2+y^2=218$

$x^2+(x-6)^2=218$

$x^2+x^2-12x+36=218$

$2x^2-12x-182=0$

$2(x^2-6x-91)=0$

$2(x-13)(x+7)=0$

$x-13=0$ or $x+7=0$

$x=13$ $x=-7$

If $x=13$, $y=x-6$

$=7$

If $x=-7$, $y=x-6$

$=-13$

The two numbers are 13 and 7 or $-7$ and $-13$.

④ 4. The difference of two numbers is 7. The difference of the squares of the numbers is 77. Find the numbers.

Let $x$ be the larger number.
Let $y$ be the smaller number.

$x-y=7$

$x=y+7$

$x^2-y^2=77$

$(y+7)^2-y^2=77$

$y^2+14y+49-y^2=77$

$14y+49=77$

$14y=28$

$y=2$

If $y=2$, $x=y+7$

$x=9$

OR

$x-y=7$

$-y=-x+7$

$y=x-7$

$x^2-(x-7)^2=77$

$x^2-\left(x^2-14x+49\right)=77$

$x^2-x^2+14x-49=77$

$14x-49=77$

$14x=126$

$x=9$

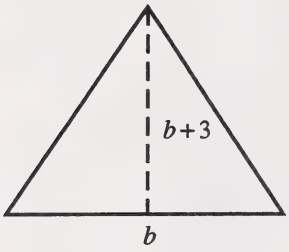
If $x=9$, $y=x-7$

$=2$

The two numbers are 9 and 2.

④ 5. The height of a triangle is 3 cm more than the base. If the area of the triangle is 20 $cm^2$, find the dimensions of the triangle.

Let $b$ be the base.
Then the height ($h$) is $b+3$.




$$A = \frac{1}{2}bh$$

$$20 = \frac{1}{2}b(b+3)$$

$$40 = b(b+3)$$

$$40 = b^2 + 3b$$

$$b^2 + 3b - 40 = 0$$

$$(b+8)(b-5) = 0$$

$b+8=0$ or $b-5=0$

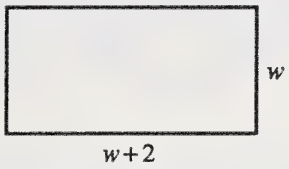
$b=-8$ $b=5$

Therefore, $b=-8$ and $b=5$.

Discard $-8$ since the base cannot have a negative measure; thus, the base is 5 cm and the height is $b+3=8$ cm.

④ 6. The length of a rectangle is 2 cm more than its width. If the area of the rectangle is 35 $cm^2$, find the dimensions of the rectangle.

Let $w$ be the width.
Then the length ($l$) will be $w+2$.




$$A = lw$$
$$35 = (w+2)(w)$$
$$(w+2)(w) = 35$$
$$w^2 + 2w - 35 = 0$$
$$(w+7)(w-5) = 0$$

$w+7=0$ or $w-5=0$
$w=-7$ $w=5$

Therefore, $w=-7$ and $w=5$.

Discard $-7$ since the width cannot be a negative number; thus, the width is 5 cm and the length is $w+2=7$ cm.

**Topic 3**

30 marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL

## UNIT 5

## COORDINATE GEOMETRY

Distance Learning

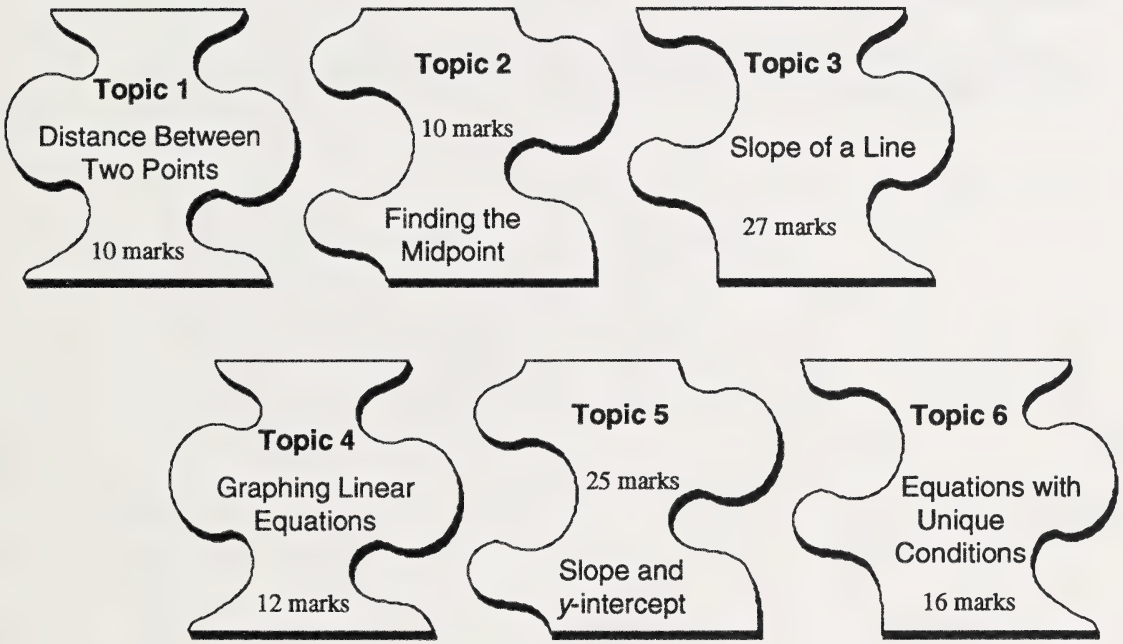
Alberta EDUCATION

## Unit 5 Overview

This unit called Coordinate Geometry is worth 19% of the course mark.

The unit consists of six topics. Each topic has an assignment found in the Assignment Booklet.

### Unit 5
### Coordinate Geometry



## Concept Introduction Possibilities

- open with concept of mass or city street systems and relate to coordinate plane
- relate slope to steepness of the roof of a house or a ski hill
- relate use of equations of lines in tracking storms

## Possible Presentation Activities

- Use a graphing calculator or a computer graphing program to plot points, graph lines, determine parallel or perpendicular lines, and determine $x$-intercepts, $y$-intercepts, and points of intersection.

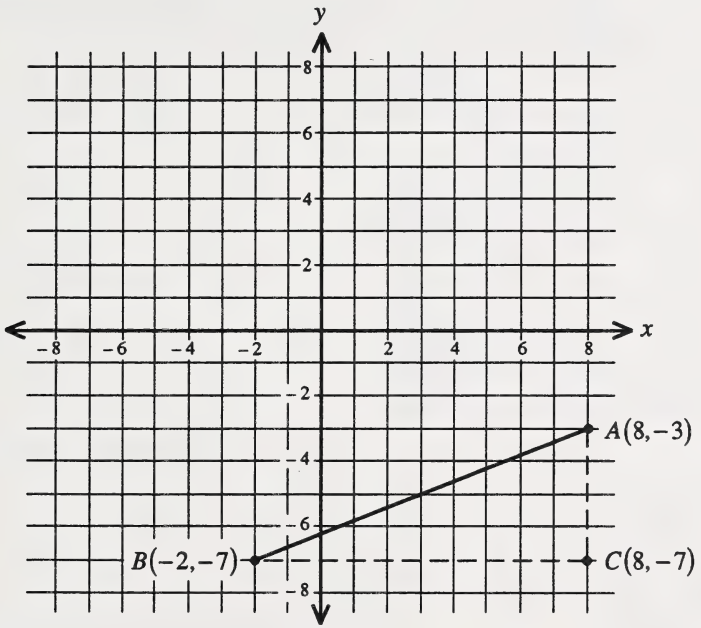
## Marking Guidelines

- If the answer is correct, give full marks whether or not any work is shown.
- If the answer is incorrect and work is shown, give partial marks for any correct work shown.

## Topic 1: Distance Between Two Points

(2)

1. $A(8,-3)$ and $B(-2,-7)$ are two points on a line. Use the Pythagorean theorem to find the distance between $A$ and $B$. (A graph will be helpful.)




$(AB)^2 = (AC)^2 + (BC)^2$

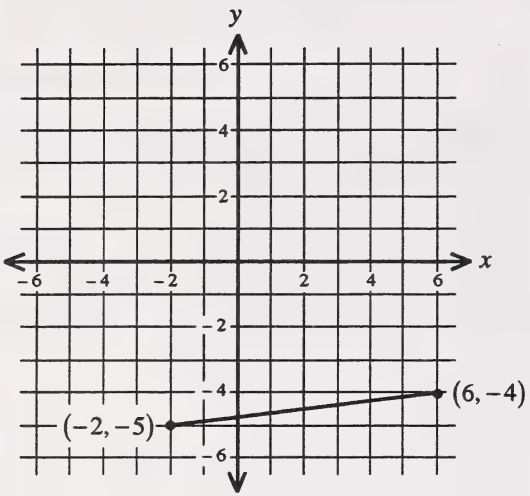
$(AB)^2 = (4)^2 + (10)^2$

$(AB)^2 = 16 + 100$

$(AB)^2 = 116$

$AB \doteq 10.8$

(3) 2. Using the distance formula, determine the length of a line segment whose endpoints are $(-2,-5)$ and $(6,-4)$.




$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-2-6)^2 + [-5-(-4)]^2}$$
$$= \sqrt{(-8)^2 + (-1)^2}$$
$$= \sqrt{64+1}$$
$$= \sqrt{65}$$
$$\doteq 8.1$$

The length of the line segment is about 8.1 units.

(2) 3. Explain how the distance formula and the Pythagorean theorem are similar.

In the distance formula, $d$ is the length of a line segment whose endpoints are on a coordinate plane. This line segment is the hypotenuse of a right triangle that can be constructed using that line segment. The Pythagorean theorem can also be used to find the length of the hypotenuse of a right triangle.

③ 3. The coordinates of the endpoints of the diameter of a circle are $(-8,-7)$ and $(4,5)$. What is the length of the radius?

Method I

The length of the diameter is as follows:

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-8-4)^2+(-7-5)^2} \\ &= \sqrt{(-12)^2+(-12)^2} \\ &= \sqrt{144+144} \\ &= \sqrt{288} \\ &\doteq 16.97 \end{aligned}$$

$$\begin{aligned} \text{Radius} &= \frac{\text{diameter}}{2} \\ &\doteq \frac{16.97}{2} \\ &\doteq 8.5 \end{aligned}$$

The radius is about 8.5 units.

Method II

The centre of the circle (or midpoint of the diameter) is as follows:

$$\begin{aligned} \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) &= \left(\frac{-8+4}{2}, \frac{-7+5}{2}\right) \\ &= \left(\frac{-4}{2}, \frac{-2}{2}\right) \\ &= (-2,-1) \end{aligned}$$

The radius is the length from the centre of the circle to one endpoint of the diameter.

$$\begin{aligned} r &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ &= \sqrt{[-8-(-2)]^2+[-7-(-1)]^2} \\ &= \sqrt{(-8+2)^2+(-7+1)^2} \\ &= \sqrt{(-6)^2+(-6)^2} \\ &= \sqrt{36+36} \\ &= \sqrt{72} \\ &= 6\sqrt{2} \\ &\doteq 8.5 \end{aligned}$$

The length of the radius is about 8.5 units.

**Topic 1**

__10__ marks

## Topic 2: Finding the Midpoint

③ 1. The midpoint of a line segment with one endpoint $(-3, 7)$ is $(5, 9)$. Find the other endpoint.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (5, 9)$$

Let $(x_1, y_1) = (-3, 7)$.

Substitute $x_1 = -3$.

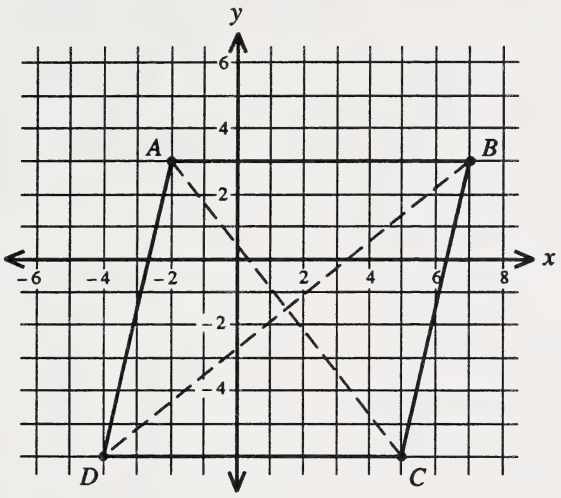
$$\frac{-3 + x_2}{2} = 5$$
$$-3 + x_2 = 10$$
$$x_2 = 13$$

Substitute $y_1 = 7$.

$$\frac{7 + y_2}{2} = 9$$
$$7 + y_2 = 18$$
$$y_2 = 11$$

The other endpoint of the line segment is $(13, 11)$.

2. The points $A(-2,3)$, $B(7,3)$, $C(5,-6)$, and $D(-4,-6)$ form a quadrilateral. Label the points on the grid and draw the diagonals.




(3) a. Find the midpoint of the diagonals. What do you notice?

The diagonals are $\overline{AC}$ and $\overline{BD}$.

$$\text{midpoint of } \overline{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+5}{2}, \frac{3-6}{2}\right) = \left(\frac{3}{2}, \frac{-3}{2}\right)$$

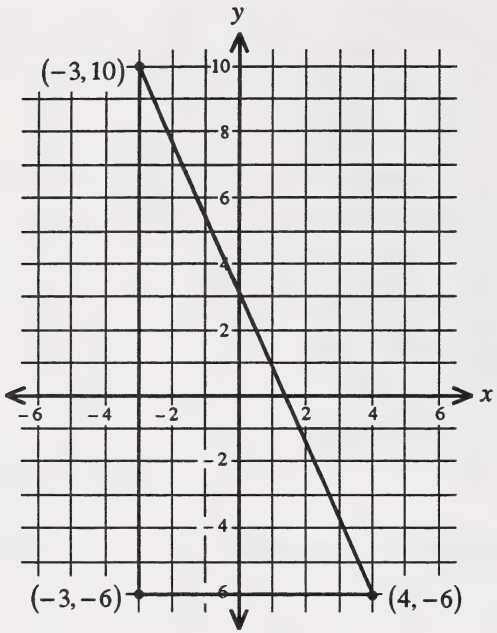
$$\text{midpoint of } \overline{BD} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{7-4}{2}, \frac{3-6}{2}\right) = \left(\frac{3}{2}, \frac{-3}{2}\right)$$

The diagonals have a common midpoint. The coordinates of the point of intersection of the diagonals are $\left(\frac{3}{2}, \frac{-3}{2}\right)$.

(2) b. Explain how you could show that the diagonals bisect each other.

Label the midpoint $E$. Use the distance formula to show that $AE = CE$ and $BE = DE$.

(2) 3. Find the midpoint of the hypotenuse of a triangle whose vertices are $(-3,-6)$, $(4,-6)$, and $(-3,10)$.




Points $(-3,10)$ and $(4,-6)$ represent the endpoints of the hypotenuse.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3+4}{2},\frac{10-6}{2}\right)$$
$$=\left(\frac{1}{2},\frac{4}{2}\right)$$
$$=\left(\frac{1}{2},2\right)$$

**Topic 2**

__10__ marks

## Topic 3: Slope of a Line

(3) 1. A line passes through the points $P(10,-6)$ and $Q(-4,k)$ and has a slope of $\frac{2}{7}$. Find $k$.

$$m=\frac{y_1-y_2}{x_1-x_2}$$

$$\frac{2}{7}=\frac{k-(-6)}{-4-10}$$

$$\frac{2}{7}=\frac{k+6}{-14}$$

$$7(k+6)=-14(2)$$

$$7k+42=-28$$

$$7k=-28-42$$

$$7k=-70$$

$$k=\frac{-70}{7}$$

$$=-10$$

(3) 2. The endpoints of $\overline{AB}$ are $A(2,9)$ and $B(-4,-7)$. The endpoints of $\overline{CD}$ are $C(6,8)$ and $D(12,24)$. Show that $\overline{AB}$ and $\overline{CD}$ are parallel.

$$m_{\overline{AB}}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-7-9}{-4-2}$$

$$=\frac{-16}{-6}$$

$$=\frac{8}{3}$$

$$m_{\overline{CD}}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{24-8}{12-6}$$

$$=\frac{16}{6}$$

$$=\frac{8}{3}$$

Since $m_{\overline{AB}}=m_{\overline{CD}}$, $\overline{AB}\parallel\overline{CD}$.

3. The vertices of a right triangle are $P(-3,10)$, $Q(11,-2)$, and $R(6,-5)$.

(3) a. Show that $\overline{PR}$ is perpendicular to $\overline{RQ}$.

$$m_{\overline{PR}} = \frac{y_1 - y_2}{x_1 - x_2} \qquad m_{\overline{RQ}} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{10-(-5)}{-3-6} \qquad = \frac{-5-(-2)}{6-11}$$

$$= \frac{15}{-9} \qquad = \frac{-3}{-5}$$

$$= -\frac{5}{3} \qquad = \frac{3}{5}$$

Since $\left(-\frac{5}{3}\right)\left(\frac{3}{5}\right) = -1$, $\overline{PR} \perp \overline{RQ}$.

(2) b. Find the midpoints of $\overline{PR}$ and $\overline{PQ}$.

$$\text{midpoint of } \overline{PR} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-3+6}{2}, \frac{10-5}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$\text{midpoint of } \overline{PQ} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-3+11}{2}, \frac{10-2}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{8}{2}\right)$$

$$= (4,4)$$

③ c. Show that the line through the midpoints is parallel to $\overline{QR}$.

$$\text{Slope of line through the midpoints} = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{\frac{5}{2} - 4}{\frac{3}{2} - 4}$$
$$= \frac{-1\frac{1}{2}}{-2\frac{1}{2}}$$
$$= \frac{3}{5}$$

$$\text{slope of } \overline{QR} = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{-2-(-5)}{11-6}$$
$$= \frac{3}{5}$$

**Note:** The slope of $\overline{QR}$ is the same as the slope of $\overline{RQ}$. Since the slope of $\overline{RQ}$ is $\frac{3}{5}$ we can also just write the slope of $\overline{QR}$ is $\frac{3}{5}$.

The slopes are equal; therefore, the lines are parallel.

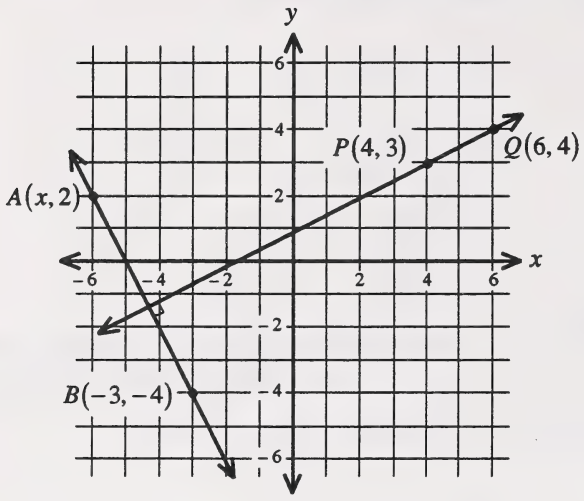
③ 4. When roads are constructed through the mountains, the engineer considers the slope of the climb. Explain why.

If the road is too steep, climbing will be difficult, especially for huge trucks with heavy loads. Icy roads in winter will further compound the problem.

② 5. A section of a ski hill has a vertical drop of 461 m over a horizontal distance of 1844 m. What is the slope of the run?

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{461}{1844}$$
$$= \frac{1}{4}$$

③ 6. Find the value of $x$ so that the line passing through $A(x,2)$ and $B(-3,-4)$ is perpendicular to the line passing through $P(4,3)$ and $Q(6,4)$.




$$m_{\overline{PQ}} = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{3-4}{4-6}$$
$$= \frac{-1}{-2}$$
$$= \frac{1}{2}$$

Since $\overline{AB}$ is perpendicular to $\overline{PQ}$, the slope of $AB = -2$.

$$\frac{y_1 - y_2}{x_1 - x_2} = m_{\overline{AB}}$$
$$\frac{2-(-4)}{x-(-3)} = -2$$
$$\frac{2+4}{x+3} = -2$$
$$-2(x+3) = 6$$
$$-2x - 6 = 6$$
$$-2x = 12$$
$$x = -6$$

(2) 7. Maziar is walking from Megamall, with coordinates $(-9,-1)$, to Minimall, with coordinates $(6,8)$. His friend Heidi asked him if he could pick up a computer program at Discount Computer House, with coordinates $(-4,2)$. Heidi said that the computer shop is on the way. Explain how Heidi can show that the computer shop is directly in line with the two malls.

Heidi can show that the computer shop is directly in line with the two malls in one of the following two ways:

- Using the slope formula, she can show that the slope of the line connecting the two malls is the same as the slope of the line that connects Megamall and the computer shop, and the line that connects Minimall and the computer shop.
- Using the distance formula, she can show that the distance between the two malls is equal to the sum of the distance between Megamall and the computer shop and the distance between Minimall and the computer shop.

(3) 8. Use the distance formula to show that points $D(5,4)$, $E(2,7)$, and $F(-2,11)$ lie on the same line.

$$d_{\overline{DE}} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(5-2)^2 + (4-7)^2}$$
$$= \sqrt{(3)^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2}$$

$$d_{\overline{EF}} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{[2-(-2)]^2 + (7-11)^2}$$
$$= \sqrt{(4)^2 + (-4)^2}$$
$$= \sqrt{16+16}$$
$$= 4\sqrt{2}$$

$$d_{\overline{DF}} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{[5-(-2)]^2 + (4-11)^2}$$
$$= \sqrt{(7)^2 + (-7)^2}$$
$$= 7\sqrt{2}$$

Since $7\sqrt{2} = 3\sqrt{2} + 4\sqrt{2}$, $D$, $E$, and $F$ are collinear.

**Topic 3**

27 marks

## Topic 4: Graphing Linear Equations

1. A clothing firm pays its employees a fixed salary of $200 per week plus a commission of $20 per suit sold.

(1) a. Write an equation showing an employee's earnings per week.

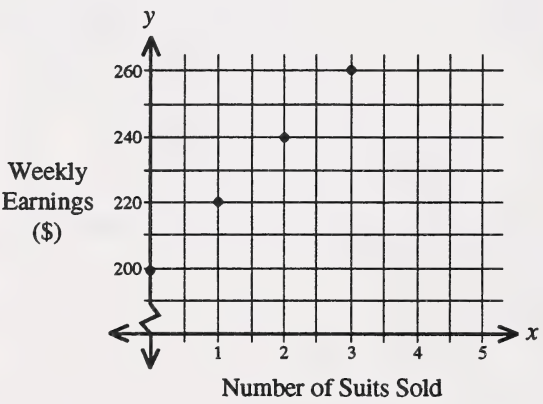
Let $y$ be the weekly earnings and $x$ be the number of suits sold.

$y = 200 + 20x$, where $x$ is a whole number

(2) b. Construct a table, and plot the points on a graph.

| $x$ | $y$ |
|---|---|
| 0 | 200 |
| 1 | 220 |
| 2 | 240 |
| 3 | 260 |

**Note:** The table of values may vary.




(1) c. Why does the graph contain only positive values?

You can not sell negative suits.

(1) d. Why does the variable $x$ in your equation represent an integer?

You cannot sell a **part** of a suit.

(1) e. How many suits have to be sold to make a weekly salary of \$520?

$$\begin{aligned} y &= 200 + 20x \\ 520 &= 200 + 20x \\ 520 - 200 &= 20x \\ 320 &= 20x \\ 16 &= x \end{aligned}$$

To make a weekly salary of \$520, sixteen suits must be sold.

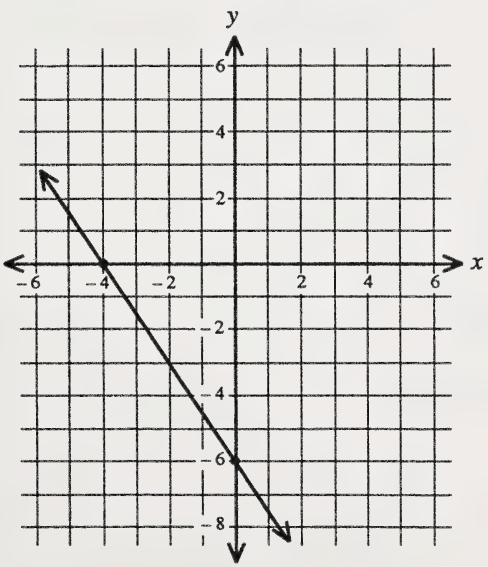
(2) 2. Use the $x$- and $y$-intercepts to graph $-3x - 2y = 12$.

Let $y = 0$.

$$\begin{aligned} -3x - 2y &= 12 \\ -3x &= 12 \\ x &= -4 \end{aligned}$$

Let $x = 0$.

$$\begin{aligned} -3x - 2y &= 12 \\ -2y &= 12 \\ y &= -6 \end{aligned}$$



3. A furniture store charges its customers \$20 for delivery within a 20 km radius and 50¢ for each additional kilometre.

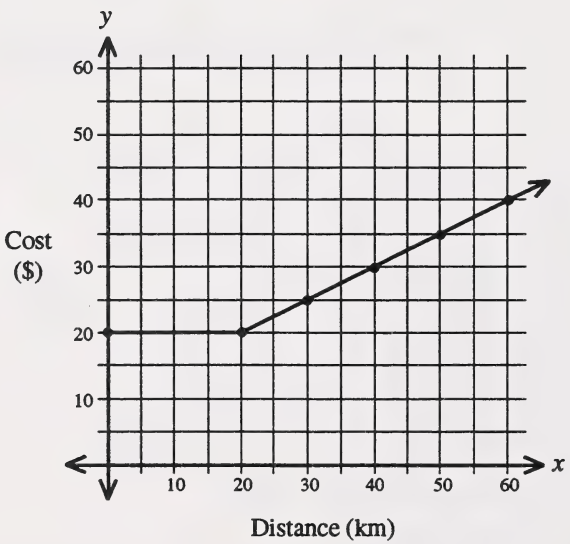
(1) a. Write an equation to show this partial variation.

Let $x$ be the number of kilometres.

$y = 20 + 0.50(x - 20)$

(1) b. Construct a table of values; then, graph the equation.

| Total trip ($x$) | $(x-20)$ | $y$ |
|---|---|---|
| 30 | 10 | 25 |
| 40 | 20 | 30 |
| 50 | 30 | 35 |
| 60 | 40 | 40 |



(1) c. If a customer is 45 km away, how much will the furniture store charge for delivery?

$$\begin{aligned} y &= 20 + 0.50(x - 20) \\ &= 20 + 0.50(45 - 20) \\ &= 20 + 0.50(25) \\ &= 20 + 12.50 \\ &= 32.50 \end{aligned}$$

The furniture store will charge $32.50.

(1) d. If a customer is 10 km away, how much will the furniture store charge for delivery?

Since the customer is less than 20 km away, the furniture store will charge $20 for delivery.

**Topic 4**

__12__ marks

## Topic 5: Slope and *y*-intercept

1. Solve for the indicated variable in terms of the other variables.

(2) a. Solve for $h$ in $m=\sqrt{3th}$.

Square both sides and reverse order.

$3th=m^2$

$h=\frac{m^2}{3t}$

(2) b. Solve for $t$ in $d=\frac{1}{2}at^2$.

Reverse order.

$\frac{1}{2}at^2=d$

Isolate $t^2$ and find the square root.

$t^2=\frac{2d}{a}$

$t=\pm\sqrt{\frac{2d}{a}}$

(3) 2. Convert the linear equation $49x+7y=49$ to $y=mx+b$ form. From this new form, determine the slope and the *y*-intercept.

$49x+7y=49$

$7y=-49x+49$

$y=-7x+7$

slope $=-7$

*y*-intercept $=7$

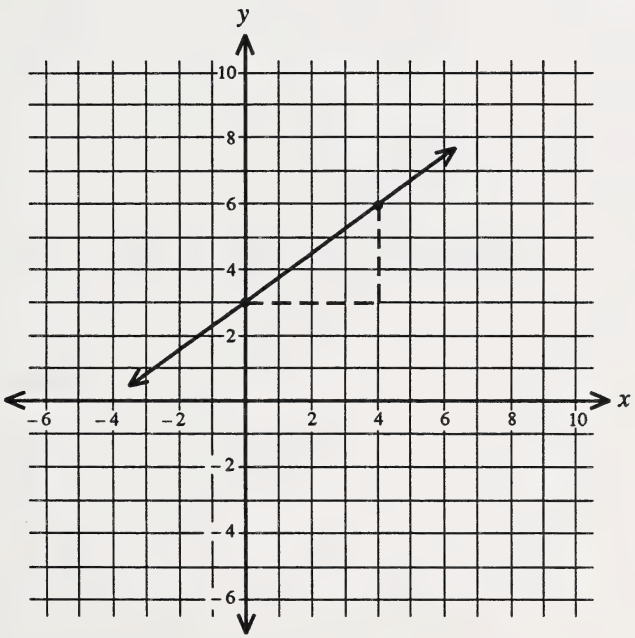
④

3. Determine the slope and the $y$-intercept of a line described by $3x-4y+12=0$. Use the slope and $y$-intercept to graph this line.

$$3x-4y+12=0$$
$$-4y=-3x-12$$
$$\frac{-4y}{-4}=\left(\frac{-3x}{-4}\right)+\left(\frac{-12}{-4}\right)$$
$$y=\frac{3}{4}x+3$$

slope $=\frac{3}{4}$

$y$-intercept $= 3$



(4) 4. Determine the equation of a line passing through point $(-3, 22)$ and having a slope of $-\frac{2}{3}$. Express the equation in $Ax + By = C$ form. Arrange the equation so that $A$ is positive. Identify the values of $A$, $B$, and $C$.

$$\frac{y-y_1}{x-x_1} = m$$

$$\frac{y-22}{x-(-3)} = -\frac{2}{3}$$

$$\frac{y-22}{x+3} = -\frac{2}{3}$$

$$-2(x+3) = 3(y-22)$$

$$-2x-6 = 3y-66$$

$$-2x-3y+60 = 0$$

$$2x+3y-60 = 0$$

$$2x+3y = 60$$

$A = 2,\ B = 3,\ C = 60$

(2) 5. The line $y = mx + 9$ contains the point $(-2, -5)$. Find the value of $m$.

$$y = mx+9$$

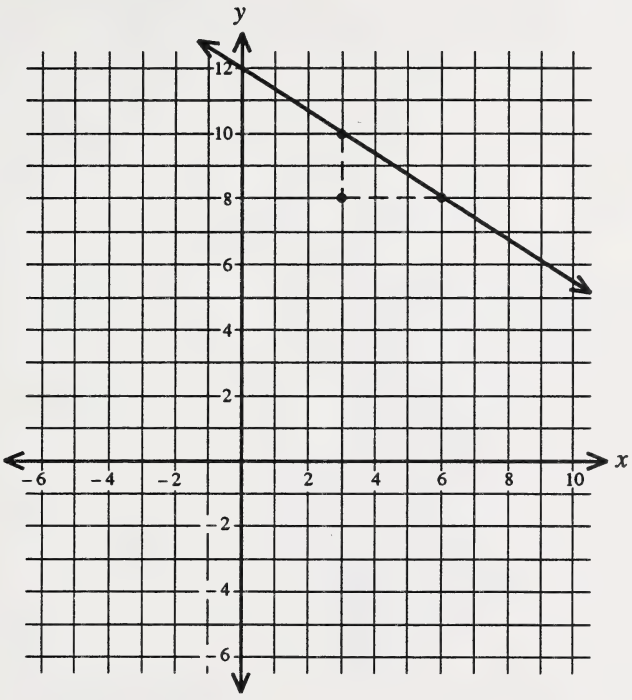
$$-5 = -2m+9$$

$$-5-9 = -2m$$

$$-14 = -2m$$

$$m = 7$$

③ 6. Draw a line with slope $-\frac{2}{3}$ through the point $(6, 8)$. Use the graph to write the equation of this line. Write the equation in standard form.




slope $m = \frac{-2}{3}$

From the graph, the $y$-intercept is 12.

$y = mx + b$

$y = \frac{-2}{3}x + 12$

In standard form:

$\frac{-2}{3}x - y + 12 = 0$

$2x + 3y - 36 = 0$

7. Graph the following lines and give the slope.

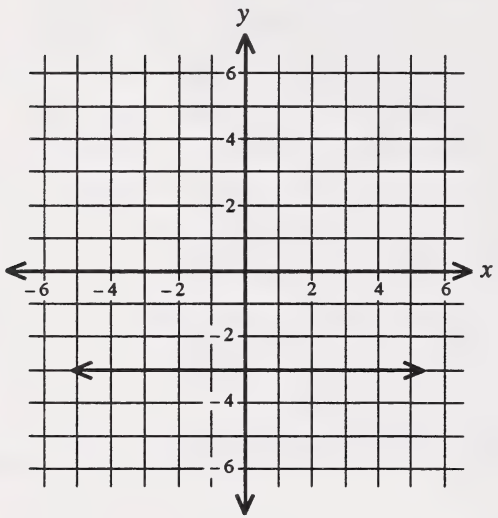
(2) a. $5y-10=-25$

$5y-10=-25$

$5y=-25+10$

$5y=-15$

$y=-3$




The slope is 0.

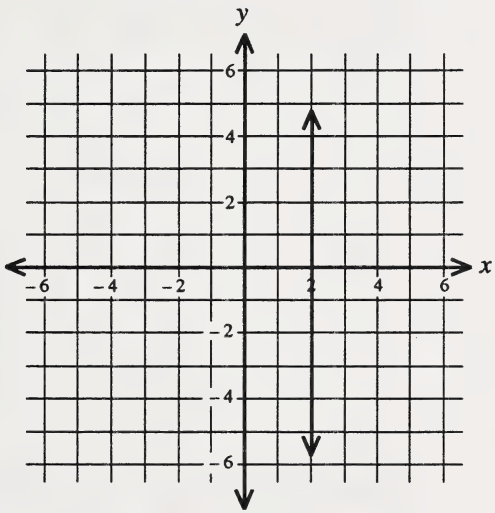
② b. $-3x+12=6$

$-3x+12=6$

$-3x=6-12$

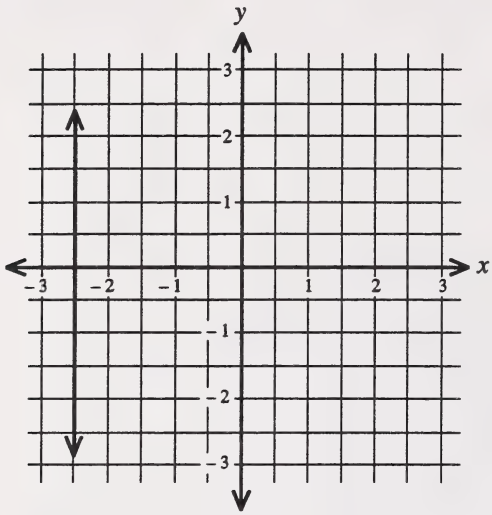
$-3x=-6$

$x=2$




The slope is undefined.

(1) 8. Write the equation for the following graph.




$x = -2.5$

**Topic 5**

__25__ marks

## Topic 6: Equations with Unique Conditions

(3) 1. Determine the equation of a line that passes through $(-1, 5)$ and is parallel to $2x-4y=3$. Write the new equation in standard form.

$$2x-4y=3$$
$$-4y=-2x+3$$
$$y=\frac{1}{2}x-\frac{3}{4}$$

$$\text{slope}=\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$
$$y-5=\frac{1}{2}(x+1)$$
$$2y-10=x+1$$
$$x-2y+11=0$$

(3) 2. Determine the equation of a line in standard form passing through $(2, 4)$, which is perpendicular to the line joining $A(-2, 6)$ and $B(-5, 8)$.

$$m_{\overline{AB}}=\frac{y_1-y_2}{x_1-x_2}$$
$$=\frac{8-6}{-5+2}$$
$$=-\frac{2}{3}$$
$$\therefore m_2=\frac{3}{2}$$

$$y-y_1=m_2(x-x_1)$$
$$y-4=\frac{3}{2}(x-2)$$
$$2y-8=3x-6$$
$$3x-2y+2=0$$

③ 3. Write the equation of a line that passes through point $(-4, 6)$ and whose $y$-intercept is equal to that for $-3x+2y=10$. Write the equation in standard form.

$$-3x+2y=10$$
$$2y=3x+10$$
$$y=\frac{3}{2}x+5$$

The $y$-intercept is 5.

Use the slope $y$-intercept form. Substitute $y=6$, $x=-4$, and $b=5$.

$$6=-4m+5$$
$$-4m=1$$
$$m=-\frac{1}{4}$$

You may also use the slope formula with points $(-4, 6)$ and $(0, 5)$.

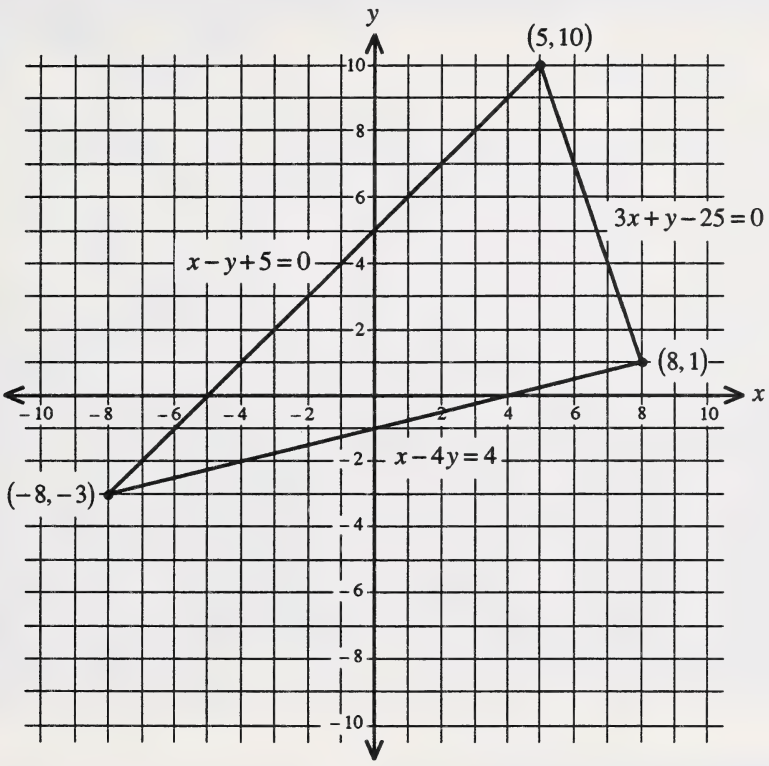
$$m=\frac{6-5}{-4-0}$$
$$=\frac{1}{-4}$$
$$=-\frac{1}{4}$$

Use the point $(-4, 6)$ and the slope of $-\frac{1}{4}$.

$$y-y_1=m(x-x_1)$$
$$y-6=-\frac{1}{4}(x+4)$$
$$4y-24=-x-4$$
$$x+4y-20=0$$

4. The equations of the three line segments are $x-y+5=0$, $3x+y-25=0$, and $x-4y=4$. Use the grid to graph these line segments. Use whichever method of graphing you find easiest for each equation.

(3) a. The line segments intersect to form a triangle. Find the coordinates of the vertices from the graph.



④ b. Determine the equation of the line in standard form joining the midpoints of the first two line segments ($x-y+5=0$ and $3x+y-25=0$).

Find the midpoint of $x-y+5=0$.

$$\left(\frac{5-8}{2},\frac{10-3}{2}\right)=\left(\frac{-3}{2},\frac{7}{2}\right)$$

Find the midpoint of $3x+y-25=0$.

$$\left(\frac{5+8}{2},\frac{10+1}{2}\right)=\left(\frac{13}{2},\frac{11}{2}\right)$$

Find the slope of the line.

$$\frac{y_2-y_1}{x_2-x_1}=\frac{\frac{7}{2}-\frac{11}{2}}{\frac{-3}{2}-\frac{13}{2}}$$

$$=\frac{\frac{-4}{2}}{\frac{-16}{2}}$$

$$=\frac{-4}{2}\times\frac{2}{-16}$$

$$=\frac{1}{4}$$

Use the point-slope form.

$$(y-y_1)=m(x-x_1)$$

$$y-\frac{7}{2}=\frac{1}{4}\left[x-\left(\frac{-3}{2}\right)\right]$$

$$4y-14=x+\frac{3}{2}$$

$$-x+4y-14-\frac{3}{2}=0$$

$$-x+4y-\frac{31}{2}=0$$

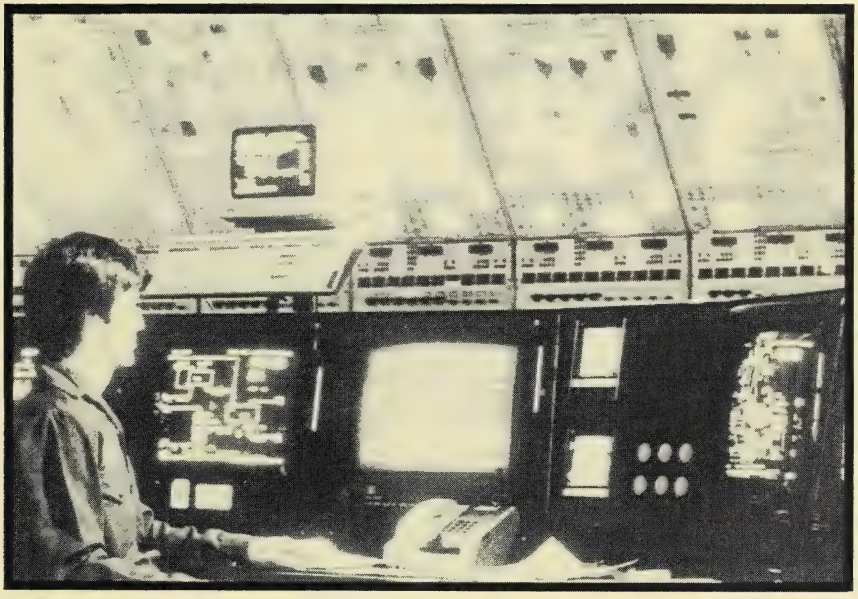
$$2x-8y+31=0$$

**Topic 6**

16 marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL



UNIT 6

## SYSTEMS of EQUATIONS

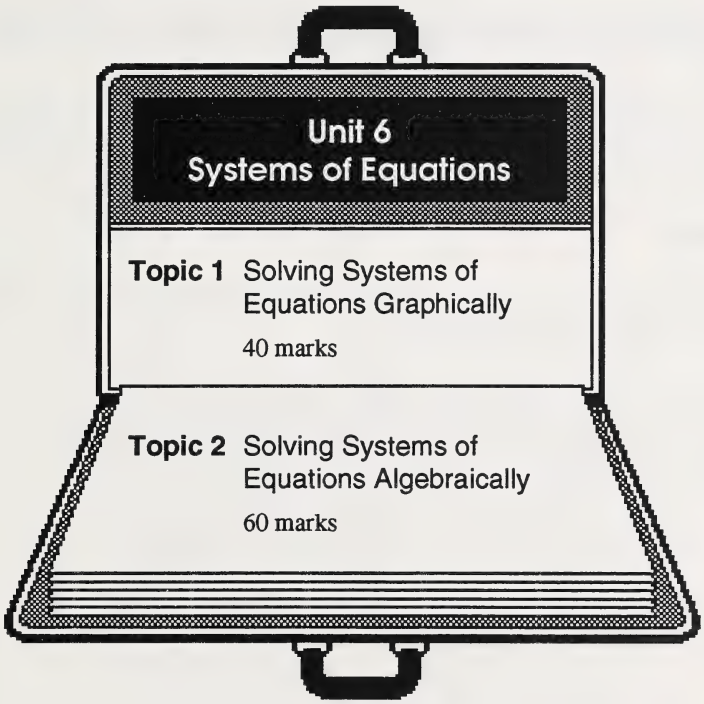
Distance Learning

Alberta EDUCATION

## Unit 6 Overview

This unit called Systems of Equations is worth 10% of the course mark.

The unit consists of two topics. Each topic has an assignment in the Assignment Booklet.



## Concept Introduction Possibilities

- Solve problem such as the following by making a table of values and graphing each line.

  The good ship *Slenderize* is on course defined by the equation $2y + x = 10$. The sloop *John Doe* is on a course defined by the equation $y = -2x + 11$. Will the paths of these two ships cross? If yes, where?

- Emphasize that the coordinates of the point of intersection of two lines is the only pair of numbers that will satisfy both equations.

- Discuss the need for algebraic methods for solving. For example, the solution for some systems of equations would be difficult to determine from a graph because they do not intersect at integral values. Estimating fractions from a graph is not an accurate method of determining a solution.

## Marking Guidelines

- When marking the solutions to the systems of equations, keep in mind that any of the three algebraic methods may be used unless specified in the instructions.

- If the answer is correct, give full marks whether or not work is shown.

- If the answer is incorrect and work is shown, give partial marks for correct work.

## Topic 1: Solving Systems of Equations Graphically

1. Solve and classify the following systems of equations by graphing.

(6)

a. $5x-3y=15$ (1)
$10x-6y=18$ (2)

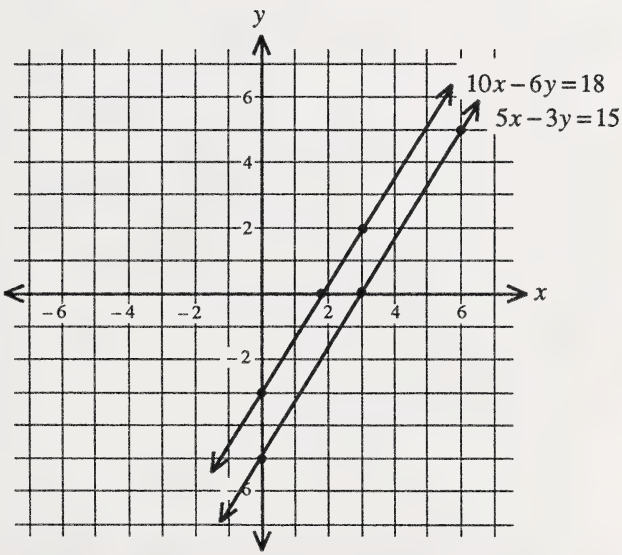
Make a table of values for each equation. (Other values may be used.)

$5x-3y=15$

| $x$ | $y$ |
|---|---|
| 3 | 0 |
| 0 | −5 |
| 6 | 5 |

$10x-6y=18$

| $x$ | $y$ |
|---|---|
| 1.8 | 0 |
| 0 | −3 |
| 3 | 2 |




The system is inconsistent. There is no solution.

⑥ b. $3x-5y=-21$ ①
$x+7y=19$ ②

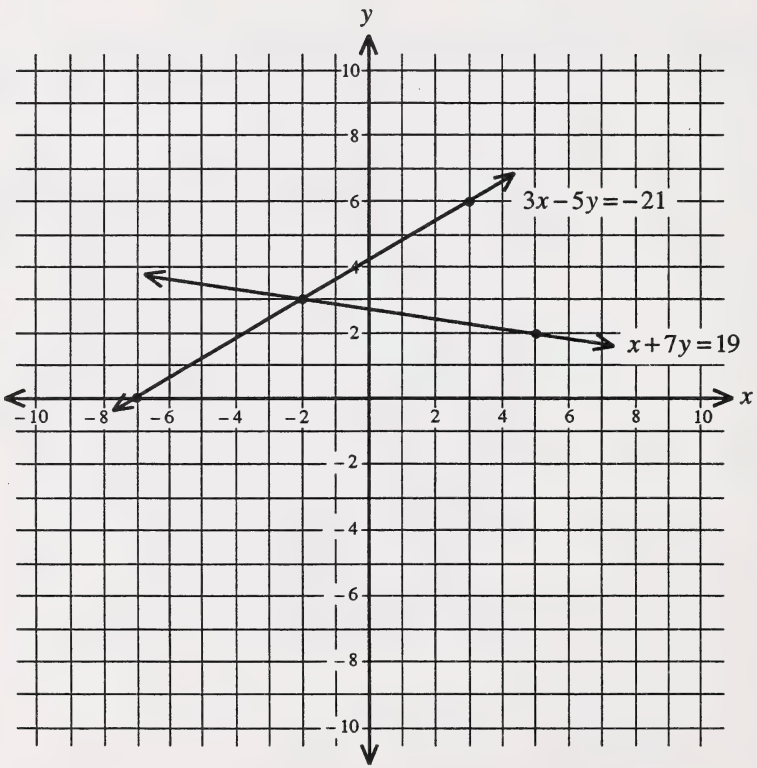
Make a table of values for each equation. (Other values may be used.)

$3x-5y=-21$

| $x$ | $y$ |
|---|---|
| $-7$ | 0 |
| $-2$ | 3 |
| 3 | 6 |

$x+7y=19$

| $x$ | $y$ |
|---|---|
| 19 | 0 |
| $-2$ | 3 |
| 5 | 2 |




The point of intersection is $(-2, 3)$. Therefore, the solution is $x=-2$ and $y=3$. The system is independent.

⑥ c. $x-3y=15$ ①

$\frac{2}{3}x-2y=10$ ②

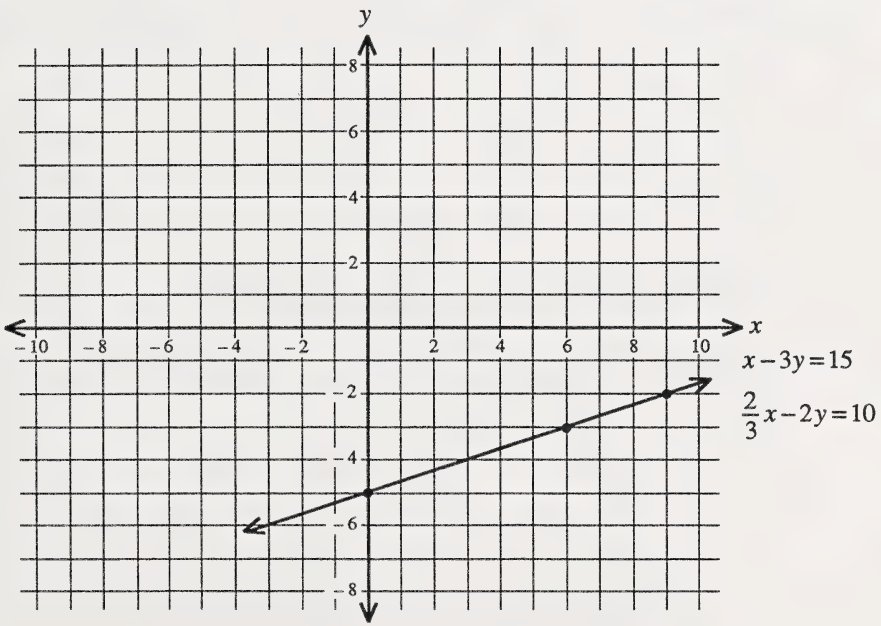
Make a table of values for each equation.

$x-3y=15$

| $x$ | $y$ |
|---|---|
| 15 | 0 |
| 9 | −2 |
| 6 | −3 |
| 0 | −5 |

$\frac{2}{3}x-2y=10$

| $x$ | $y$ |
|---|---|
| 15 | 0 |
| 9 | −2 |
| 6 | −3 |
| −6 | −7 |




Both equations are on the same line. This is a dependent system.

(8) 2. Classify the following system of equations as dependent, independent, or inconsistent according to the slopes and the $y$-intercepts of the equations.

$5x-10y=11$ (1)

$6x-12y=\frac{66}{5}$ (2)

From (1): $-10y=-5x+11$

$$\frac{-10y}{-10}=\frac{-5x}{-10}+\frac{11}{-10}$$

$$y=\frac{1}{2}x-\frac{11}{10}$$

For (1), the slope $=\frac{1}{2}$ and the $y$-intercept $=-\frac{11}{10}$.

From (2): $-12y=-6x+\frac{66}{5}$

$$\frac{-12y}{-12}=\frac{-6x}{-12}+\frac{\frac{66}{5}}{-12}$$

$$y=\frac{1}{2}x-\frac{66}{60}$$

$$y=\frac{1}{2}x-\frac{11}{10}$$

For (2), the slope $=\frac{1}{2}$ and the $y$-intercept $=-\frac{11}{10}$.

The slopes and the $y$-intercepts of the two equations are equal. Thus, the system is dependent.

(8) 3. If a number is added to $\frac{1}{3}$ of a second number, the sum is 6. If $\frac{2}{3}$ of the second number is subtracted from $\frac{1}{2}$ of the first number, the difference is $-2$. Use the method of graphing to find these two numbers.

Let $x$ be the first number.
Let $y$ be the second number.

$x+\frac{1}{3}y=6$ (1)

$\frac{1}{2}x-\frac{2}{3}y=-2$ (2)

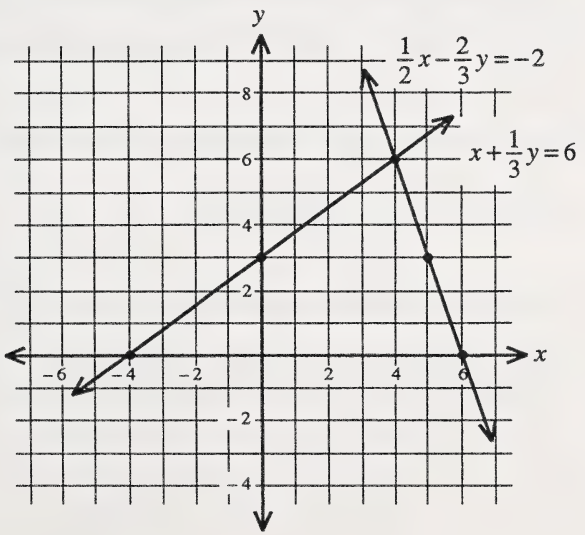
Simplify the equations; then, make a table of values for each question.

(1)$\times 3$: $3x+y=18$

| $x$ | $y$ |
|---|---|
| 6 | 0 |
| 0 | 18 |
| 5 | 3 |
| 4 | 6 |

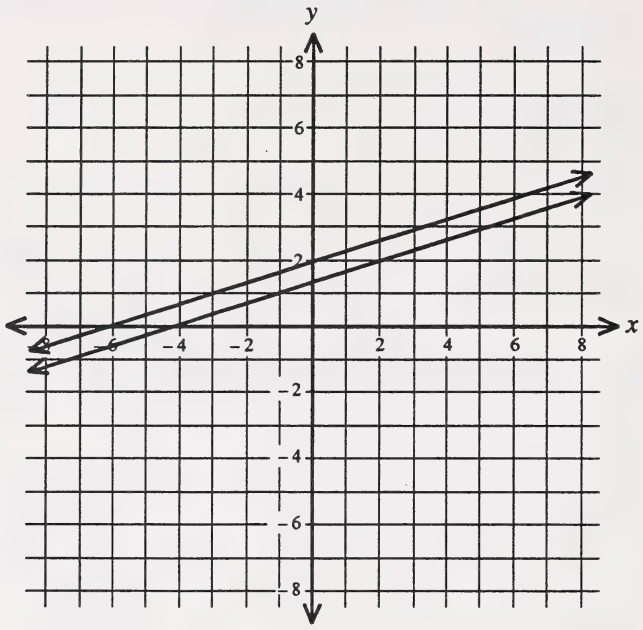
(1)$\times 6$: $3x-4y=-12$

| $x$ | $y$ |
|---|---|
| $-4$ | 0 |
| 0 | 3 |
| 4 | 6 |




The first number is 4 and the second number is 6.

4. Answer questions 4a and 4b using the following graph.




(2) a. Determine the solution to the system of equations represented by the lines on the graph.

There is no value for $x$ and $y$ that is a solution to this system of equations.

(2) b. How does the graph show that this is the solution to this system of equations?

The lines do not intersect or coincide.

(2) c. What types of graphs give real-number solutions?

Graphs which have lines that intersect or coincide give real-number solutions.

**Topic 1**

40 marks

## Topic 2: Solving Systems of Equations Algebraically

1. Solve each of the following systems of equations by any algebraic method. Then, classify the system.

(7) a. $7x-6y=15$ (1)
$5x-3y=12$ (2)

(2)$\times 2$: $10x-6y=24$ (3)
$7x-6y=15$ (1)
(3)$-$(1): $3x-0y=9$
$3x=9$
$x=3$

Substitute $x=3$ into (2).

$5(3)-3y=12$
$15-3y=12$
$-3y=-3$
$y=1$

The solution is $x=3$ and $y=1$. This is an independent system.

(7) b. $2x+6y=11$ (1)
$\frac{1}{3}x+y=\frac{11}{6}$ (2)

(2)$\times 6$: $2x+6y=11$ (3)
$2x+6y=11$ (1)
(3)$-$(1): $0x+0y=0$
$0=0$

There are an infinite number of solutions. This is a dependent system.

(7) c. $\frac{2}{3}x-\frac{1}{3}y=2$ (1)

$\frac{3}{2}x+\frac{1}{2}y=7$ (2)

$(1)\times 3$: $2x-y=6$ (3)

$(2)\times 2$: $3x+y=14$ (4)

$(3)+(4)$: $5x+0y=20$

$5x=20$

$x=4$

Substitute $x=4$ into (4).

$3(4)+y=14$

$y=14-12$

$=2$

The solution is $x=4$ and $y=2$. This is an independent system.

(7) d. $y=-2x+3$ (1)

$2x+y-1=0$ (2)

From (1): $2x+y=3$ (3)

From (2): $2x+y=1$ (4)

$2x+y=3$ (3)

$2x+y=1$ (4)

$(3)-(4)$: $0x+0y=2$

$0+0=2$

$0=2$

Since $0\neq 2$, there is no solution. This is an inconsistent system.

2. Translate the following verbal expressions into algebraic expressions.

(2) a. Two Mathematics 10 text books and five dictionaries cost $55.

$2x+5y=55$

(3) b. One-half the sum of two numbers is fifteen more than one-third their differences.

$\frac{1}{2}(x+y)=15+\frac{1}{3}(x-y)$

(8) 3. A table and four chairs cost $245. Next week, the price of the table will increase by 15% and the price of the chairs will increase by 10%. The same table and chairs will cost $273.75. What is the present cost of the table? What is the present cost of each chair?

Let $x$ be the cost of the table.
Let $y$ be the cost of a chair.

$x+4y=245$ (1)

$1.15x+1.1(4y)=273.75$

$1.15+4.4y=273.75$ (2)

(1) $\times 1.15$: $1.15x+4.6y=281.75$ (3)

$1.15x+4.4y=273.75$ (2)

(3) $-$ (2): $0x+0.2y=8$

$0.2y=8$

$y=40$

Substitute $y=40$ into (1).

$x+4(40)=245$

$x+160=245$

$x=85$

The table costs $85 and the chairs cost $40 each.

(3) 4. Which method of solving equations (substitution, comparison, or elimination) is easiest to apply to the following system of equations. Explain.

$2x+3y=8$

$5x-2y=-3$

The elimination method would be the easiest to apply. The substitution method is not easy because some of the coefficients that are substituted for the first variable are not integers. The comparison method is only used when the coefficients of either $x$ or $y$ in both equations are the same.

(8) 5. There is a two-digit number. The units digit exceeds the tens digit by two. Two times the tens digit exceeds the units digit by five. Find this two-digit number.

Let $x$ be the tens digit.
Let $y$ be the units digit.

$y-x=2$ (1)

$2x=y+5$ (2)

From (1): $y=x+2$ (3)

Substitute $y=x+2$ in (2).

$2x=x+2+5$

$2x=x+7$

$x=7$

Substitute $x=7$ into (3).

$y=7+2$

$y=9$

The number is 79.

(8) 6. It takes George $\frac{3}{4}$ h to row his boat 3 km upstream and $\frac{1}{2}$ h to return. Find the speed of the boat in still water and the speed of the current.

Let $x$ be the speed of the boat.
Let $y$ be the speed of the current.

$$(x-y)\times\frac{3}{4}=3$$

$$\frac{3}{4}x-\frac{3}{4}y=3 \quad (1)$$

$$(x+y)\times\frac{1}{2}=3$$

$$\frac{1}{2}x+\frac{1}{2}y=3 \quad (2)$$

$$(1)\times 4:\ 3x-3y=12 \quad (3)$$
$$(2)\times 6:\ 3x+3y=18 \quad (4)$$
$$(3)+(4):\ 6x+0y=30$$
$$6x=30$$
$$x=5$$

Substitute $x=5$ into (3).

$$3(5)-3y=12$$
$$15-3y=12$$
$$-3y=-3$$
$$y=1$$

The speed of the boat is 5 km/h and the speed of the current is 1 km/h.

**Topic 2**

60 marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL

## UNIT 7

## TRIGONOMETRY

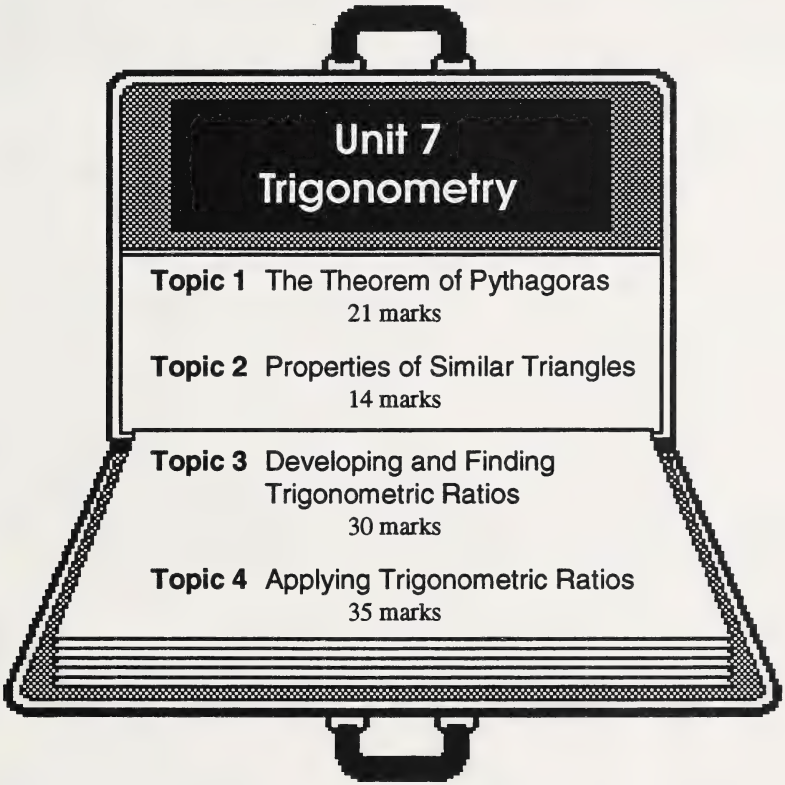
Distance Learning

Alberta EDUCATION

## Unit 7 Overview

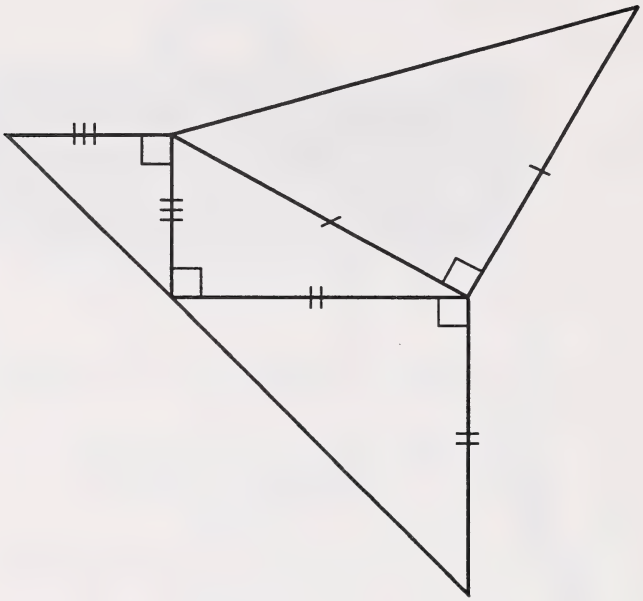
This unit called Trigonometry is worth 10% of the course mark.

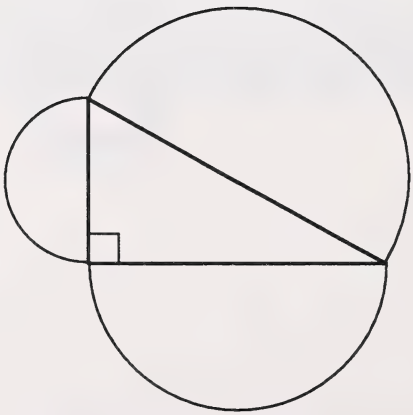
The unit consists of four topics. Each topic has an assignment in the Assignment Booklet.



## Concept Introduction Possibilities

Demonstrate the Pythagorean theorem using shapes other than squares on the sides of the right triangle. These shapes may be right triangles, semicircles, or equilateral triangles. Find the area of the different shapes to show that the theorem holds for other shapes as well as squares. Look at the following diagrams for examples.





## Marking Guidelines

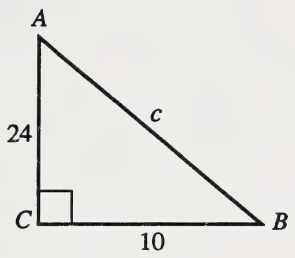
- If the answer is correct, give full marks whether or not work is shown.
- If the answer is incorrect and work is shown, give partial marks for correct work.

## Topic 1: The Theorem of Pythagoras

1. Use the Pythagorean theorem to solve for the unknown in each right triangle. Round to two decimal places where necessary.

(3)

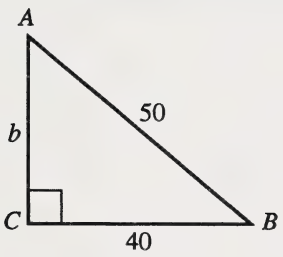
a.




$$c^2 = a^2 + b^2$$
$$c^2 = (10)^2 + (24)^2$$
$$c^2 = 100 + 576$$
$$c^2 = 676$$
$$\sqrt{c^2} = \sqrt{676}$$
$$c = 26$$

(3)

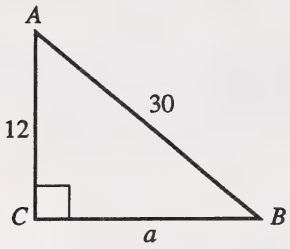
b.




$$c^2 = a^2 + b^2$$
$$(50)^2 = (40)^2 + b^2$$
$$2500 = 1600 + b^2$$
$$b^2 = 2500 - 1600$$
$$b^2 = 900$$
$$\sqrt{b^2} = \sqrt{900}$$
$$b = 30$$

③

c.

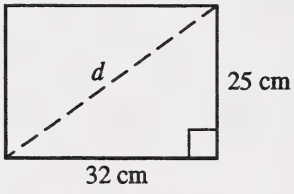



$$c^2 = a^2 + b^2$$
$$(30)^2 = a^2 + (12)^2$$
$$900 = a^2 + 144$$
$$a^2 = 900 - 144$$
$$a^2 = 756$$
$$\sqrt{a^2} = \sqrt{756}$$
$$a \doteq 27.495\,454\,17$$
$$\doteq 27.50$$

④

2. The screen size of a computer monitor is the measure of the diagonal. This diagonal is a line segment joining the opposite vertices of a rectangle. If the screen measures 25 cm by 32 cm, what is the screen's size? Round your answer to the nearest centimetre.

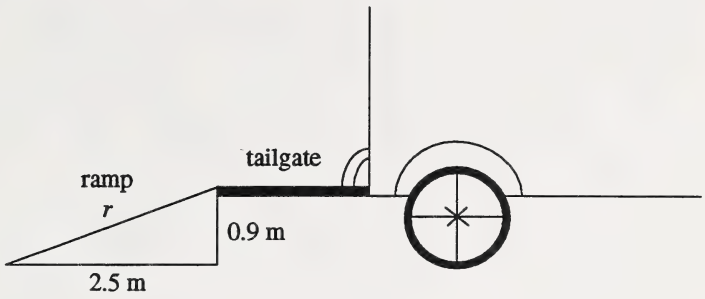
$d$ = diagonal




$$d^2 = (25)^2 + (32)^2$$
$$d^2 = 625 + 1024$$
$$\sqrt{d^2} = \sqrt{1649}$$
$$d \doteq 40.607\,881\,01$$
$$\doteq 41$$

The size of the computer screen is approximately 41 cm.

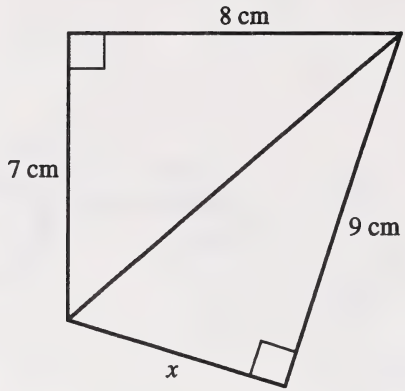
④ 3. The tailgate of a truck is 0.9 m above the ground. What is the length of a ramp needed to reach a point on the ground 2.5 m away? Round your answer to the nearest tenth of a metre.




$$r^2 = (0.9)^2 + (2.5)^2$$
$$r^2 = 0.81 + 6.25$$
$$r^2 = 7.06$$
$$\sqrt{r^2} = \sqrt{7.06}$$
$$r \doteq 2.657\,066\,051$$
$$\doteq 2.7 \text{ m}$$

The ramp needed must be approximately 2.7 m in length.

④ 4. Solve for $x$. Round your answer to one decimal place.




Find the hypotenuse of the larger triangle first.

$$h^2 = (8)^2 + (7)^2$$
$$h^2 = 64 + 49$$
$$h^2 = 113$$
$$\sqrt{h^2} = \sqrt{113}$$
$$h = \sqrt{113}$$

The solve for $x$ using the other triangle.

$$\left(\sqrt{113}\right)^2 = x^2 + 9^2$$
$$x^2 = 113 - 81$$
$$x^2 = 32$$
$$\sqrt{x^2} = \sqrt{32}$$
$$x \doteq 5.656\,854\,249$$
$$\doteq 5.7 \text{ cm}$$

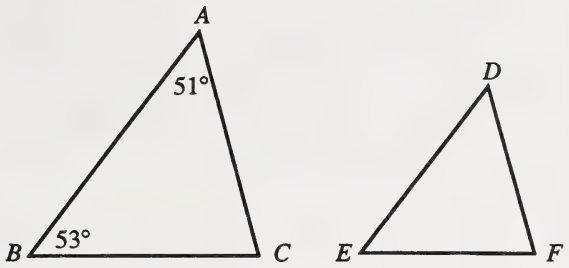
The value for $x$ is approximately 5.7 cm.

**Topic 1**

__21__ marks

## Topic 2: Properties of Similar Triangles

(2) 1. $\Delta ABC \sim \Delta DEF$



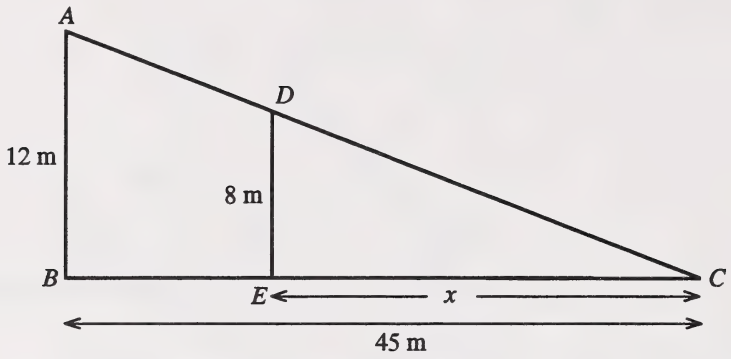

$\angle C =$ **76°** The sum of the angles in a triangle is 180°.

$\angle D =$ **51°**

$\angle E =$ **53°** } Corresponding angles of similar triangles are equal.

$\angle F =$ **76°**

③ 2. Find the value of $x$.




Since $\Delta ABC \sim \Delta DEC$, corresponding sides are proportional.

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{12}{8} = \frac{45}{x}$$

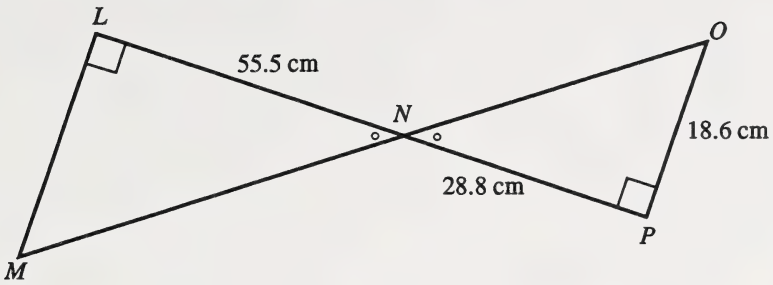
$$12x = 360$$

$$\frac{12x}{12} = \frac{360}{12}$$

$$x = 30 \text{ m}$$

The value of $x$ is 30 m.

3. Use the diagram to answer the following questions.




(3) a. Find the length of $\overline{LM}$ rounded to one decimal place.

$$\frac{LM}{PO} = \frac{LN}{PN}$$
$$\frac{LM}{18.6} = \frac{55.5}{28.8}$$
$$28.8(LM) = 55.5(18.6)$$
$$28.8(LM) = 1032.3$$
$$\frac{28.8(LM)}{28.8} = \frac{1032.3}{28.8}$$
$$LM = 35.843\,75$$
$$\doteq 35.8$$

The length of $\overline{LM}$ is approximately 35.8 cm.

(3) b. Find the length of $\overline{MN}$ rounded to one decimal place.

$$(MN)^2 = (LM)^2 + (LN)^2$$
$$(MN)^2 \doteq (35.8)^2 + (55.5)^2$$
$$(MN)^2 \doteq 1281.64 + 3080.25$$
$$(MN)^2 \doteq 4361.89$$
$$\sqrt{(MN)^2} \doteq \sqrt{4361.89}$$
$$MN \doteq 66.044\,606\,14$$
$$\doteq 66.0$$

The length of $\overline{MN}$ approximately is 66.0 cm.

(3) c. Find the length of $\overline{NO}$ rounded to one decimal place.

$$\frac{NO}{NM} = \frac{PN}{LN}$$

$$\frac{NO}{66.0} \doteq \frac{28.8}{55.5}$$

$$55.5(NO) \doteq 28.8(66.0)$$

$$\frac{55.5(NO)}{55.5} \doteq \frac{1900.8}{55.5}$$

$$NO \doteq 34.248\,648\,565$$

$$\doteq 34.2$$

The length of $\overline{NO}$ is 34.2 cm.

Another way to find the length of $\overline{NO}$ is as follows:

$$(NO)^2 = (OP)^2 + (NP)^2$$

$$(NO)^2 = (18.6)^2 + (28.8)^2$$

$$(NO)^2 = 345.96 + 829.44$$

$$(NO)^2 = 1175.4$$

$$\sqrt{(NO)^2} = \sqrt{1175.4}$$

$$NO \doteq 34.284\,107\,11$$

$$\doteq 34.3$$

The length of $\overline{NO}$ is 34.3 cm.

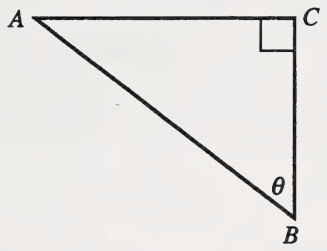
The difference in the results is due to rounding.

**Topic 2**

__14__ marks

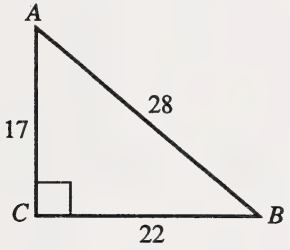
## Topic 3: Developing and Finding Trigonometric Ratios

(3) 1. Name the hypotenuse, the side opposite, and the side adjacent to $\angle\theta$ in $\Delta ABC$.




The hypotenuse is $c$ or $\overline{AB}$.
The side opposite is $b$ or $\overline{AC}$.
The side adjacent is $a$ or $\overline{CB}$.

(12) 2. Give the primary trigonometric ratios for each acute angle in $\Delta ABC$. Change each fractional ratio to a decimal numeral rounded to four decimal places.




For $\angle A$, the ratios are as follows:

$$\sin A = \frac{22}{28} = \frac{11}{14} \doteq 0.7857 \qquad \cos A = \frac{17}{28} \doteq 0.6071 \qquad \tan A = \frac{22}{17} \doteq 1.2941$$

For $\angle B$, the ratios are as follows:

$$\sin B = \frac{17}{28} \doteq 0.6071 \qquad \cos B = \frac{22}{28} = \frac{11}{14} \doteq 0.7857 \qquad \tan B = \frac{17}{22} \doteq 0.7727$$

(3) 3. Referring to question 2, explain how you can find the sine and cosine ratios for angle $B$ without calculating once you have calculated the sine and cosine ratios for angle $A$.

Since $\sin A = \cos B$ and $\cos A = \sin B$, simply write in the appropriate value.

4. Use a calculator to give the angle measure for each of the following. Round your answers to the nearest degree.

(2) a. $\tan A = 2.0345$

$\angle A \doteq 63.824\,902\,24$

$\doteq 64°$

(2) b. $\cos B = \frac{3}{7}$

$\cos B \doteq 0.428\,571\,428$

$\angle B \doteq 64.623\,066\,48$

$\doteq 65°$

(2) c. $\sin C = 0.0398$

$\angle C \doteq 2.280\,974\,488$

$\doteq 2°$

(4) 5. a. Using a calculator, find the sine and cosine ratios for the following four angles. Round the ratios to four decimal places.

| Angle | 76° | 63° | 29° | 7° |
|---|---|---|---|---|
| Sine ratio | **0.9703** | **0.8910** | **0.4848** | **0.1219** |
| Cosine ratio | **0.2419** | **0.4540** | **0.8746** | **0.9925** |

(1) b. What happens to the value of the sine ratios as the measure of the angle decreases?

The sine ratio decreases as the measure of the angle decreases.

(1) c. What happens to the value of the cosine ratios as the measure of the angle decreases?

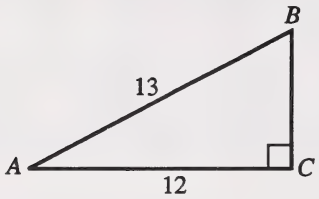
The cosine ratio increases as the measure of the angle decreases.

**Topic 3**

__30__ marks

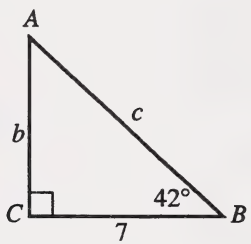
## Topic 4: Applying Trigonometric Ratios

(2) 1. Given $\Delta ABC$, find $\angle A$ to the nearest degree.




$$\cos A = \frac{12}{13}$$
$$\cos A \doteq 0.923\,076\,923$$
$$\angle A \doteq 22.619\,864\,95$$
$$\doteq 23°$$

(2) 2. Given $\Delta ABC$, determine the length of $b$ rounded to the nearest tenth.

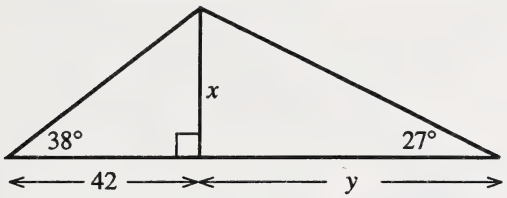



$$\tan 42° = \frac{b}{7}$$
$$b = 7(\tan 42°)$$
$$\doteq 7(0.900\,404\,044)$$
$$\doteq 6.302\,828\,31$$
$$\doteq 6.3 \text{ units}$$

3. Find each unknown indicated. Round each answer to one decimal place.

④

a.




To find $x$, use the following:

$$\tan 38° = \frac{x}{42}$$

$$x = 42(\tan 38°)$$

$$\doteq 42 \times 0.781\,285\,626$$

$$\doteq 32.813\,996\,31$$

$$\doteq 32.8 \text{ units}$$

To find $y$, use the following:

$$\tan 27° \doteq \frac{32.8}{y}$$

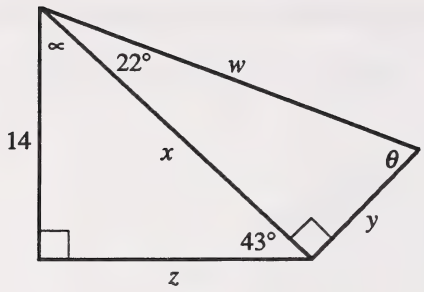
$$y \doteq \frac{32.8}{\tan 27°}$$

$$\doteq \frac{32.8}{0.509\,525\,449}$$

$$\doteq 64.373\,624\,58$$

$$\doteq 64.4 \text{ units}$$

⑥ b.




$\theta = 180° - (90° + 22°)$

$= 180° - 112°$

$= 68°$

$\propto = 180° - (90° + 43°)$

$= 180° - 133°$

$= 47°$

To find $x$, use the following:

$$\sin 43° = \frac{14}{x}$$

$$x = \frac{14}{\sin 43°}$$

$$\doteq \frac{14}{0.681\,998\,36}$$

$$\doteq 20.527\,9086$$

$$\doteq 20.5 \text{ units}$$

To find $y$, use the following:

$$\tan 22° \doteq \frac{y}{20.5}$$

$$y \doteq 20.5(\tan 22°)$$

$$\doteq 20.5 \times 0.404\,026\,225$$

$$\doteq 8.282\,537\,63$$

$$\doteq 8.3 \text{ units}$$

To find $w$, use the following:

$$w^2 \doteq (20.5)^2 + (8.3)^2$$

$$w^2 \doteq 420.25 + 68.89$$

$$w^2 \doteq 489.14$$

$$\sqrt{w^2} \doteq \sqrt{489.14}$$

$$w \doteq 22.116\,509\,67$$

$$\doteq 22.1 \text{ units}$$

To find $z$, use the following:

$$(20.5)^2 \doteq z^2 + (14)^2$$

$$420.25 \doteq z^2 + 196$$

$$z^2 \doteq 420.25 - 196$$

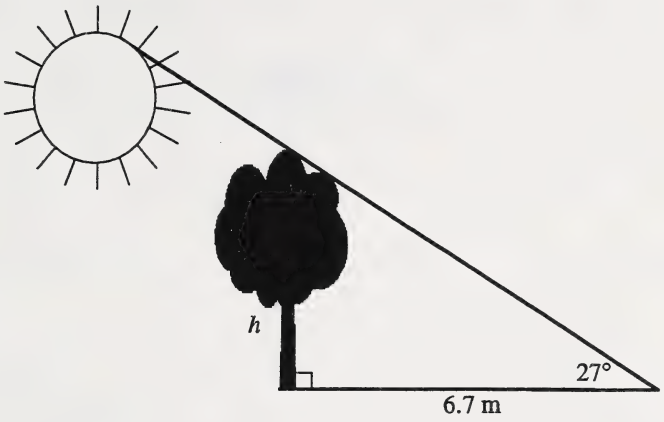
$$z^2 \doteq 224.25$$

$$\sqrt{z^2} \doteq \sqrt{224.25}$$

$$z \doteq 14.974\,979\,13$$

$$\doteq 15.0 \text{ units}$$

③ 4. As the sun sets in the west, a tree casts a shadow 6.7 m long. The angle of elevation of the sun is 27°. Find the height of the tree. Round your answer to the nearest tenth of a metre.




$$\tan 27° = \frac{h}{6.7}$$

$$h = 6.7(\tan 27°)$$

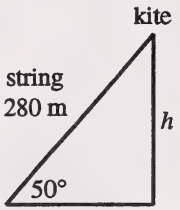
$$\doteq 6.7 \times 0.509\,525\,449$$

$$\doteq 3.413\,820\,512$$

$$\doteq 3.4 \text{ m}$$

The height of the tree is approximately 3.4 m.

(3) 5. The length of a string attached to a kite is 280 m. The angle that the string makes with the ground is 50°. At what height is the kite flying? Round your answer to the nearest metre.




$$\sin 50° = \frac{h}{280}$$

$$h = 280(\sin 50°)$$

$$\doteq 280 \times 0.766\,044\,443$$

$$\doteq 214.492\,4441$$

$$\doteq 214 \text{ m}$$

The kite is flying approximately 214 m above the ground.

(3) 6. A steep road in the mountains rises 8.4 m for every 30.3 m of road. What is the angle of elevation of the steep, mountain road? Round your answer to the nearest degree.

$$\sin \theta = \frac{8.4}{30.3}$$

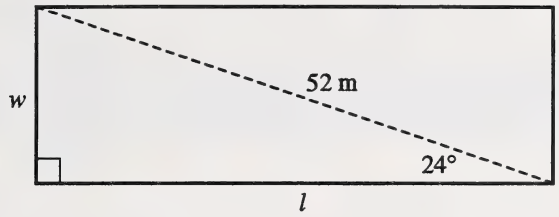
$$\sin \theta \doteq 0.277\,227\,722$$

$$\theta \doteq 16.094\,815\,99$$

$$\doteq 16°$$

The angle of elevation of the road is approximately 16°.

⑥ 7. Use the information in the following diagram to find the area of the rectangle. Round the measures of $l$ and $w$ to the nearest metre.




To find $l$, use the following:

$$\cos 24° = \frac{l}{52}$$
$$l = 52(\cos 24°)$$
$$\doteq 52 \times 0.913\,545\,457$$
$$\doteq 47.504\,3638$$
$$\doteq 48 \text{ m}$$

The length of the rectangle is approximately 48 m.

To find $w$, use the following:

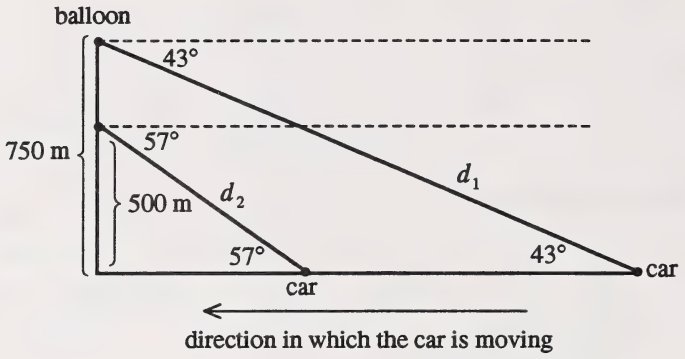
$$\sin 24° = \frac{w}{52}$$
$$w = 52(\sin 24°)$$
$$\doteq 52 \times 0.406\,736\,643$$
$$\doteq 21.150\,305\,44$$
$$\doteq 21 \text{ m}$$

The width of the rectangle is approximately 21 m.

$$A = lw$$
$$\doteq 48 \times 21$$
$$\doteq 1008 \text{ m}^2$$

The area of the rectangle is approximately 1008 $\text{m}^2$.

(6) 8. From a hot-air balloon 750 m above the ground, the angle of depression of a car is 43°. The pilot took a second reading when the balloon was at an altitude of 500 m. This time the angle of depression was 57°. What was the difference in direct distance from the balloon to the moving car when the two readings were taken? Round your answer to the nearest metre.




To find $d_1$, use the following:

$$\sin 43° = \frac{750}{d_1}$$

$$d_1 = \frac{750}{\sin 43°}$$

$$\doteq \frac{750}{0.681\,998\,36}$$

$$\doteq 1099.709\,389$$

To find $d_2$, use the following:

$$\sin 57° = \frac{500}{d_2}$$

$$d_2 = \frac{500}{\sin 57°}$$

$$\doteq \frac{500}{0.838\,670\,568}$$

$$\doteq 596.181\,6464$$

The difference in the direct distance is as follows:

$$1099.709\,389 - 596.181\,646\,4 \doteq 503.527\,7428$$

$$\doteq 504 \text{ m}$$

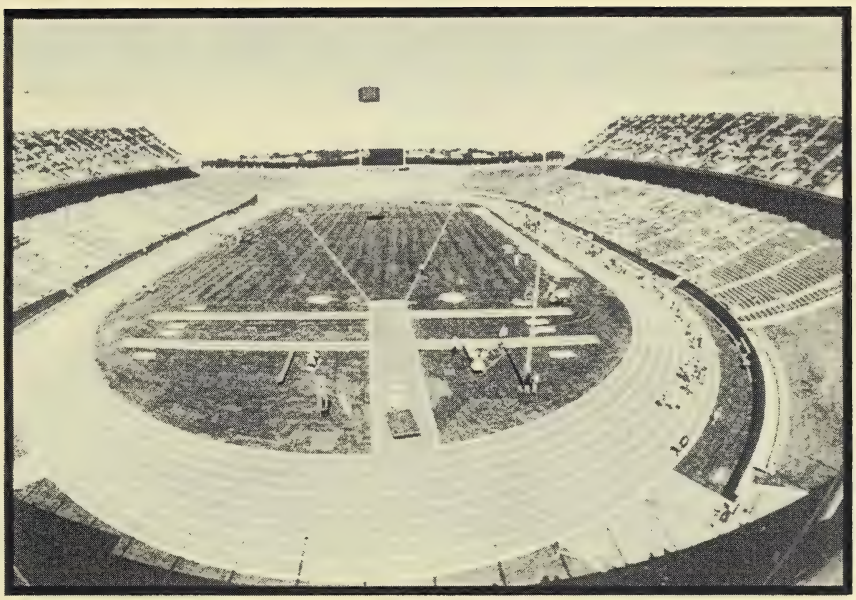
The difference in the direct distance is approximately 504 m.

**Topic 4**

__35__ marks

# MATHEMATICS 10

LEARNING FACILITATOR'S MANUAL



UNIT 8

## STATISTICS

Distance Learning

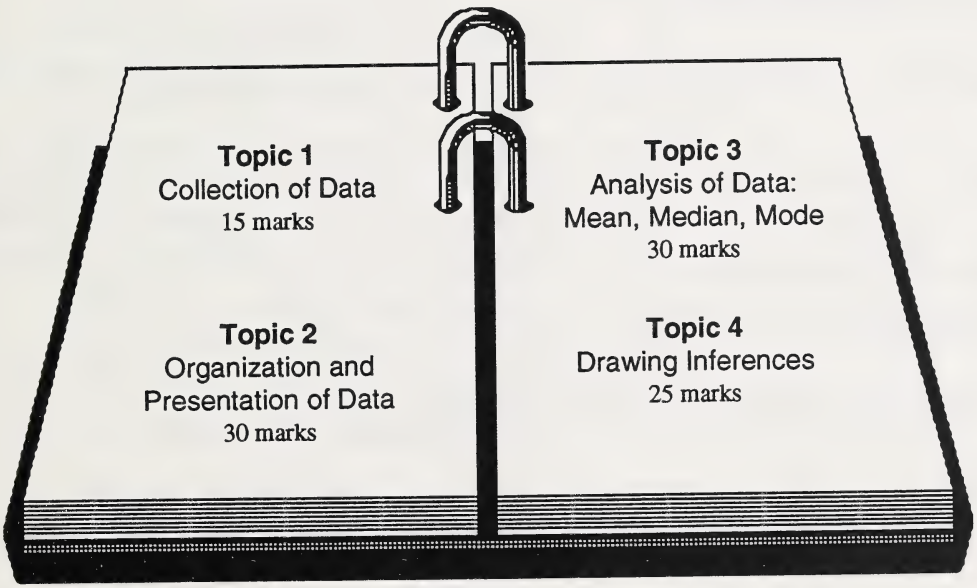
Alberta EDUCATION

## Unit 8 Overview

This unit called Statistics is worth 14% of the course mark.

The unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.

### Unit 8 Statistics



## Concept Introduction Possibilities

- Discuss sampling methods used for such surveys as TV polls and political polls.
- Discuss how a sample may be biased.
- Carry out a classroom survey such as asking students how many hours of TV they watched the previous night. Discuss whether or not the results could be applied to the whole school, the town or city, or the whole country.

## Possible Presentation Activities

- Collect graphs and charts from magazines and newspapers. Have students present them to the class. They should try to answer questions regarding sample size, method of selection, and possible bias.
- Give the students a set of topics for a survey. See page 102 of the Teacher Resource Manual for some possibilities. Then the Students, individually or in groups of two or three, can choose a topic, select a sampling method, determine the size of the sample, and carry out the survey. They can then organize and display the data using one of the methods discussed. They can later present their findings to the class and answer questions from their classmates and the teacher.

## Marking Guidelines

- If the answer is correct, give full marks whether or not any work is shown.
- If the answer is incorrect and work is shown, give partial marks for any correct work shown.

## Topic 1: Collection of Data

(1) 1. For the following question, circle the correct response.

Imagine that you want to find the average weight of a Canadian high school student. You decide to weigh every high school student in all of Calgary, Regina, and Kelowna. The **sample** of your population is

A. every high school student in Canada

B. any students in Calgary, Regina, and Kelowna

C. all the students in Canada

(D.) every high school student in Calgary, Regina, and Kelowna

(2) 2. Suppose there is to be a federal election and a poll is taken to determine which political party is favoured by the people of Canada. It is decided that 1000 people from each of the thirty most populated cities across the country will be selected at random from the phone book and asked which political party they will support in the coming election.

Is this considered a **biased** sample? Explain your answer.

This is a biased sample because it does not include areas with smaller populations. The sample should include people from towns, villages, and rural areas.

(1) 3. Suppose there is to be a provincial election and a survey is taken to determine which political party is favoured by the people of Alberta. It is decided that every third person shopping at a local store in Red Deer, Alberta is to be asked which political party they will support in the coming election.

Is this an appropriate type of sample to determine which political party is favoured in Alberta? Explain your answer.

No, this is not an appropriate type of sample because people in other areas of the province are not part of the sample.

(1) 4. For the following question, circle the correct response. Explain your answer.

Suppose you want to determine which TV program is watched most often in the province of British Columbia. Which sampling method will work best to help you determine this?

A questionnaire is sent randomly to people selected from

A. the town of Osoyoos

(B.) various provincial cities

C. the city of Victoria

D. Vancouver Island

This is the best choice because you want to determine which TV program is watched most often in the whole province, not in just one area of the province.

(1) 5. Suppose there is to be a provincial election and a survey is taken to determine which political party is favoured by the people of British Columbia. It is decided that members of a steelworkers union will be randomly selected and asked which political party they will support in the coming election.

Is this an appropriate type of sample to determine which political party is favoured in British Columbia? Give a reason for your answer.

No, this is not an appropriate type of sample because members of a steelworkers union might be partial toward one party or another. For example, while they were on strike, they might have been forced back to work by a certain political party. A survey designed to determine what the whole population favours needs to sample all parts of the population.

(1) 6. Suppose you need to find the average amount of money a family living in Edmonton spends on food in one month. To solve this problem, which sampling method will most accurately predict the average amount of money spent?

Collect food receipts from all the

(A.) grocery stores

B. fast-food restaurants

C. gasoline service stations

D. expensive restaurants

(4) 7. Suppose you were asked to collect enough data to determine the average shoe size of forty randomly selected people. Explain where, when, and how you would conduct the survey.

Answers may vary. An example is shown as follows.

Ask at least forty people to give their first name and their shoe size, and record their responses.

Choose a shoe store or the shoe department of a department store as the location for the survey. Interview people in the ladies', mens' and childrens' store departments.

Choose a Friday evening or a Saturday to ensure a more representative sample of the population. If people refuse to answer, interview more people until you get forty names and responses.

(4) 8. Explain how to use a random number table to pick any four days from all of the days in a year.

Let the numbers 000 to 364 represent the days.

Pick a spot in the random number table to start. If the three digits are from 000 to 364, then you have one day selected. If the digits are from 365 to 999, then no day has been selected.

Go to the next group of three digits. Be sure to follow in order rather than jump around the table.

Continue these steps until you have four numbers between 000 and 365. These numbers will represent the days. For example, 020 would be the twentieth day, 109 would be the one hundred and ninth day, and so on.

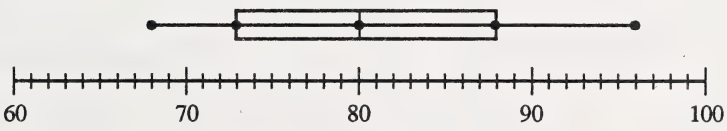
**Topic 1**

__15__ marks

## Topic 2: Organization and Presentation of Data

(5) 1. The total number of points scored by the Taber Turtlebacks in a twenty-four-game season is presented as follows. Construct a box-and-whisker plot and analyse the data by stating the median, the quartiles, and the extremes.

| | | | | | |
|---|---|---|---|---|---|
| 68 | 70 | 70 | 70 | 72 | [72] |
| [74] | 74 | 74 | 76 | 76 | (80) |
| (80) | 82 | 84 | 86 | 86 | [88] |
| [88] | 88 | 92 | 94 | 96 | 96 |



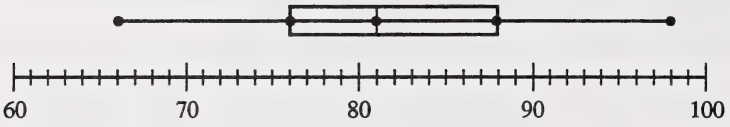

$$\text{Lower quartile} = \frac{72+74}{2}$$
$$= \frac{146}{2}$$
$$= 73$$

$$\text{Upper quartile} = \frac{88+88}{2}$$
$$= \frac{176}{2}$$
$$= 88$$

$$\text{Median} = \frac{80+80}{2}$$
$$= \frac{160}{2}$$
$$= 80$$

The extremes are 68 and 96.

(3) 2. The total number of points scored by the Wainwright Waisters in a forty-five-game season is presented by the box-and-whisker plot that follows.




a. What percentage of the population is included in the box?

The percentage of the population included in the box is 50%.

b. All of the values are between **66** and **98** .

c. What is the median of these values?

The median is 81.

(4) 3. A basketball team scored 89 points in a single game. The top four players on the team are shown with their respective number of points for the game.

| Player | Number of Points |
|---|---|
| Duncan D. | 16 |
| Jaybar A. | 8 |
| Birdie B. | 17 |
| Johnson M. | 18 |

Circle the correct response. Given the preceding data, determine which graphing technique would **best visually represent** the data. Then, draw the graph using this technique and be sure to explain the advantage of using this particular type of representation.

(A.) pictograph

B. bar graph

C. circle graph or pie chart

D. stem-and-leaf plot

Points Scored in a Basketball Game

Legend: Each ● represents two points.

Pictographs are used for making comparisons of similar data.

**Note**: A bar graph would be acceptable as well.

④ 4. Suppose a survey of several Canadian homes revealed the following numbers of boxes of breakfast cereal consumed each week. Organize this data into a frequency table. What type of graph is most appropriate for this data? Explain why you chose this particular type of graph.

| | | | | |
|---|---|---|---|---|
| 8 | 9 | 2 | 2 | 1 |
| 0 | 8 | 2 | 6 | 2 |
| 4 | 1 | 8 | 8 | 5 |
| 8 | 2 | 6 | 2 | 8 |
| 8 | 8 | 5 | 7 | 0 |
| 9 | 1 | 3 | 0 | 1 |
| 4 | 8 | 0 | 6 | 9 |
| 8 | 0 | 7 | 2 | 1 |

Number of Boxes of Cereal Consumed per Week

| Number of Boxes | Tally | Frequency |
|---|---|---|
| **0** | 𝍸 | **5** |
| **1** | 𝍸 | **5** |
| **2** | 𝍸 II | **7** |
| **3** | I | **1** |
| **4** | II | **2** |
| **5** | II | **2** |
| **6** | III | **3** |
| **7** | II | **2** |
| **8** | 𝍸 𝍸 | **10** |
| **9** | III | **3** |

Student's response may include a pictograph, bar graph, histogram, or frequency polygon. Each of these types of graphs are appropriate for comparing similar data.

③

5. Draw a pictograph to visually represent the set of data given. You are free to select your own pictograph symbol provided it relates to the nature of the data given.

A survey of forty Canadian households revealed the following number of beds found in each home.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 7 | 7 | 7 | 6 | 7 | 4 | 4 |
| 4 | 4 | 3 | 4 | 6 | 6 | 7 | 8 | 7 | 5 |
| 7 | 7 | 6 | 6 | 3 | 4 | 2 | 2 | 1 | 1 |
| 3 | 1 | 4 | 2 | 6 | 7 | 6 | 7 | 7 | 6 |

Frequency Table

| Number of Beds | Tally | Frequency |
|---|---|---|
| **1** | III | **3** |
| **2** | IIII | **4** |
| **3** | III | **3** |
| **4** | 𝍸 IIII | **9** |
| **5** | I | **1** |
| **6** | 𝍸 III | **8** |
| **7** | 𝍸 𝍸 I | **11** |
| **8** | I | **1** |

Number of Beds per Canadian Household

1 bed

2 beds

3 beds

4 beds

5 beds

6 beds

7 beds

8 beds

Legend: represents two households

③ 6. A computer has randomly generated the following even numbers. Organize the data into a stem-and-leaf plot.

| | | | | |
|---|---|---|---|---|
| 72 | 90 | 60 | 36 | 50 |
| 88 | 72 | 76 | 36 | 86 |
| 66 | 40 | 76 | 82 | 48 |
| 42 | 62 | 50 | 44 | 50 |
| 30 | 82 | 40 | 46 | 36 |
| 36 | 72 | 90 | 72 | 54 |
| 30 | 62 | 90 | 38 | 90 |
| 68 | 90 | 48 | 86 | 70 |

Correct Order

| Stem | Leaf |
|---|---|
| 3 | 0 0 6 6 6 6 8 |
| 4 | 0 0 2 4 6 8 8 |
| 5 | 0 0 0 4 |
| 6 | 0 2 2 6 8 |
| 7 | 0 2 2 2 2 6 6 |
| 8 | 2 2 6 6 8 |
| 9 | 0 0 0 0 0 |

Another Example of Correct Order

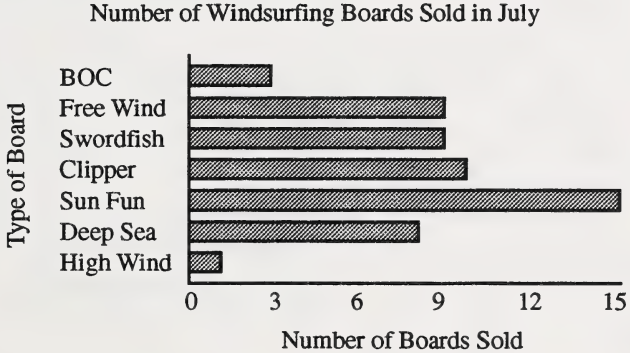
| Stem | Leaf |
|---|---|
| 3 | 0 6 0 6 6 8 6 |
| 4 | 2 0 0 8 4 6 8 |
| 5 | 0 0 0 4 |
| 6 | 6 8 2 2 0 |
| 7 | 2 2 2 6 6 2 0 |
| 8 | 8 2 2 6 6 |
| 9 | 0 0 0 0 0 |

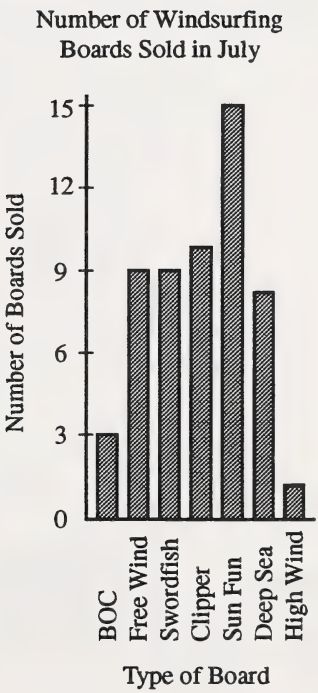
③ 7. Using a bar graph, accurately represent the data given.

Several stores in California that sell sporting goods were surveyed to determine how many windsurfing boards were sold during July, 1991. The results of this survey are shown in the following table.

| Type of Board | Frequency |
|---|---|
| BOC | 3 |
| Free Wind | 9 |
| Swordfish | 9 |
| Clipper | 10 |
| Sun Fun | 15 |
| Deep Sea | 8 |
| High Wind | 1 |

The graph may be drawn horizontally or vertically.



(5)

8. The top four goal scorers on a local hockey team are shown with their respective number of goals for the season. A total of 167 goals were scored by the entire team all season. All of the other players scored a total of 44 goals.

Complete the table and draw a circle graph to accurately represent the data. Be sure your graph is properly labelled and proportioned. Explain why the circle graph is a good choice to represent this data.

| Player | Number of Goals | Percentage (rounded to the nearest whole number) | Degrees (rounded to the nearest whole number) |
|---|---|---|---|
| Tina | 31 | **19%** | **68°** |
| Arthur | 26 | **16%** | **58°** |
| Ashley | 31 | **19%** | **68°** |
| Monique | 35 | **21%** | **76°** |
| Others | 44 | **26%** | **94°** |
| | Totals | **101%** | **364°** |

Top Goal Scorers

Due to rounding, the percentages total 101% which results in an excess of 4° since the degree measures total 364°.

The circle graph is a good choice to represent this data because you are trying to compare each of the player's scores to the total number of goals scored. Circle graphs are good for comparing the parts to the whole.

**Topic 2**

30 marks

## Topic 3: Analysis of Data: Mean, Median, Mode

(1) 1. For the following question, circle the correct response.

In shoe sales, the person responsible for ordering the number of new shoes is interested in knowing which size is selling the best. Which of the following shows this?

A. mean

(B.) mode

C. median

D. range

(1) 2. Determine the mode for the following data.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 | 13 | 9 | 16 | 9 | 6 | 14 | 12 | 7 |
| 10 | 12 | 8 | 8 | 14 | 15 | 11 | 14 | 16 |

Rearrange the data in ascending order. Although this is not necessary, it makes it easier to find the mode.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 8 | 8 | 9 | 9 | 10 | 11 | 11 |
| 12 | 12 | 13 | 14 | 14 | 14 | 15 | 16 | 16 |

The mode is 14.

(2) 3. A group of six numbers has an arithmetic mean of 8. If each of the six numbers is increased by 7, what is the new arithmetic mean?

$$\text{Mean} = \frac{6\times 8+6\times 7}{6}$$
$$= \frac{48+42}{6}$$
$$= \frac{90}{6}$$
$$= 15$$

The new arithmetic means is 15.

(2) 4. Determine the range of the given data.

| | | | | | | |
|---|---|---|---|---|---|---|
| 46.50 | 45.50 | 46.10 | 45.30 | 46.30 | 46.70 | 47.90 |
| 46.00 | 47.00 | 47.40 | 47.50 | 46.40 | 47.50 | 47.30 |

Rearrange the data in ascending order.

| | | | | | | |
|---|---|---|---|---|---|---|
| 45.30 | 45.50 | 46.00 | 46.10 | 46.30 | 46.40 | 46.50 |
| 46.70 | 47.00 | 47.30 | 47.40 | 47.50 | 47.50 | 47.90 |

$\text{Range} = 47.90 - 45.30$

$= 2.60$

The range is 2.60.

(2) 5. Determine the median of the following data.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 4 | 14 | 1 | 8 | 10 | 14 | 11 | 18 |
| 17 | 10 | 14 | 10 | 18 | 18 | 8 | 16 | |

1, 4, 5, 8, 8, 10, 10, 10, $\boxed{11}$, 14, 14, 14, 16, 17, 18, 18, 18

The median is 11.

(2) 6. Determine the mean of the following data. If necessary, express your answer to two decimal places.

| | | | | | | |
|---|---|---|---|---|---|---|
| −6 | 1 | 5 | −7 | −5 | −10 | −7 |
| −5 | 1 | 5 | 7 | −6 | −10 | 7 |

$$\text{Mean} = \frac{-6+1+5+(-7)+(-5)+(-10)+(-7)+(-5)+1+5+7+(-6)+(-10)+7}{14}$$

$$= \frac{-30}{14}$$

$$\doteq -2.142\,857\,143$$

$$\doteq -2.14$$

The mean is $-2.14$ rounded to two decimal places.

(1) 7. Complete the following statement with the correct term.

The middle term in a list of numbers arranged in ascending order is called the ___**median**___ .

(2) 8. A group of eight numbers has a mode of 25. If each of the eight numbers is decreased by 9, what is the new mode of the group of numbers?

Mode = 25 − 9
= 16

The new mode is 16.

(1) 9. Complete the following statement using the correct term or terms.

The mode and the **mean or median** are measures of central tendency.

(3) 10. The following table shows the number of ladies' shoes sold in the Villa Shoe Store in one day.

| Size | 5 | $5\frac{1}{2}$ | 6 | $6\frac{1}{2}$ | 7 | $7\frac{1}{2}$ | 8 | $8\frac{1}{2}$ | 9 | $9\frac{1}{2}$ | 10 | $10\frac{1}{2}$ | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 2 | 1 | 4 | 4 | 9 | 7 | 5 | 4 | 2 | 3 | 2 | 1 | 1 |

a. Which measure of central tendency is the best to report the average shoe size sold that day?

The mode is the best.

b. Explain why you chose that measure.

The mode is best because the average should be one of the actual values.

c. Calculate the measure.

Mode = 9

(3) 11. The following table shows the monthly salaries at Villa Shoe Store.

| | | | | |
|---|---|---|---|---|
| $1400 | $1200 | $1300 | $2900 | $1200 |
| $1550 | $1000 | $1400 | $1500 | $1150 |

a. Which measure of central tendency is the best to report the average monthly salary?

The median is the best.

b. Explain why you chose that measure.

The median is the best measure because one of the values is extreme.

c. Calculate the measure.

1000, 1150, 1200, 1200, 1300, 1400, 1400, 1500, 1550, 2900

$$\text{Median} = \frac{1300+1400}{2}$$
$$= 1350$$

The median salary is $1350.

(3) 12. The following table shows the percentage scores on a math exam.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 95 | 39 | 96 | 86 | 59 | 58 | 84 | 68 | 77 | 46 | 51 | 89 | 70 | 37 |

a. Which measure of central tendency is the best to report the average mark on the exam?

The mean is the best.

b. Explain why you chose that measure.

The mean is the best measure of average, since there are no extreme values.

c. Calculate the measure.

$$\bar{x} = \frac{37+39+46+\ldots 96}{14}$$
$$\bar{x} = \frac{955}{14}$$
$$\bar{x} \doteq 68.2$$

The average mark on the exam was about 68%.

④ 13. Determine the mean, the median, the mode, and the range for the data used in the stem-and-leaf plot shown. Round the mean to one decimal place.

| Stem | Leaf |
|---|---|
| 2 | 5 5 8 |
| 3 | 2 3 4 4 5 9 |
| 4 | 6 7 7 |
| 5 | 4 |
| 6 | 1 [1 3] 8 ← median |
| 7 | 0 1 3 4 4 5 [8 8 8] ← mode |
| 8 | 2 3 5 5 |

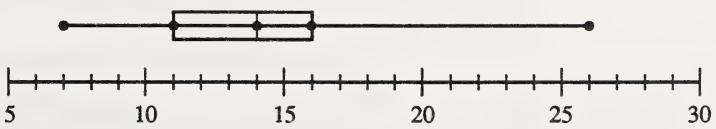
$$\text{Mean} = \frac{\text{sum of all elements}}{30}$$
$$= \frac{1738}{30}$$
$$\doteq 57.933\,333\,33$$
$$\doteq 57.9$$

$$\text{Median} = \frac{61+63}{2}$$
$$= \frac{124}{2}$$
$$= 62$$

Mode = 78

Range = 85 − 25
= 60

③ 14. Using the box-and-whisker plot shown, determine the median, the range, and the lower quartile for the data.




Median = 14

Range = 26 − 7
= 19

Lower quartile = 11

**Topic 3**

30 marks

## Topic 4: Drawing Inferences

(6)

1. An office manager has two secretaries but can only afford to keep one of them. The manager decides to keep a record of their individual typing speeds, in words per minute, and uses this to make a decision.

   Over a period of two weeks, the manager collected the following data.

   Secretary *A*:

   51 48 52 57 54
   53 57 54 56 60

   Secretary *B*:

   47 52 54 60 42
   60 50 59 73 70

   If you were the office manager, determine which of the secretaries you would keep. To do this, construct a back-to-back stem-and-leaf plot comparing the two groups of data. Also determine the median for each secretary. Based on your graph and the median, indicate which secretary will remain employed. Be sure to explain your reasons for your choice of either Secretary *A* or Secretary *B*.

   Typing Speed

| Secretary *A* | | Secretary *B* |
|---|---|---|
| 8 | 4 | 2 7 |
| 1 2 3 (4) (4) 6 7 7 | 5 | 0 2 (4) (9) |
| 0 | 6 | 0 0 |
| | 7 | 0 3 |

$$\text{Median} = \frac{54+54}{2} = \frac{108}{2} = 54$$

$$\text{Median} = \frac{54+59}{2} = \frac{113}{2} = 56.5$$

   The median for Secretary *A* is 54.

   The median for Secretary *B* is 56.5.

   Based on this information, Secretary *B* should remain employed. Secretary *B* is also showing more improvement.

(3) 2. Fill in the blanks.

a. The probability of tossing a coin and getting heads or tails is **1**.

b. The probability of picking a green marble from a jar containing red marbles is **0**.

c. The probability of picking a card with the number 3 on it, from a deck of 52 cards, is $\frac{4}{52} = \frac{1}{13}$.

(3) 3. A survey asked 2000 people what their favourite TV show is. The results are as follows.

| | |
|---|---|
| Bobsy Show | 33% |
| 57 Minutes | 27% |
| Resigning Women | 17% |
| Bash | 13% |
| Other | 10% |

In a city of 500 000 people, how many viewers would you predict will be watching the Bobsy Show this week?

$$\frac{33}{100} \times 500\,000 = 165\,000$$

You would predict that 165 000 people will be watching the Bobsy Show this week.

(2) 4. For the following question, answer **true** or **false**.

During the evening news, it is announced that the probability it will rain tomorrow is 45%. Someone listening to the broadcast could then conclude that the probability it will not rain tomorrow is $P(no\ rain) = \frac{11}{20}$.

The conclusion is **true**.

(2) 5. For the following question, circle the correct response.

The probability that an event will occur may be represented by the ratio of two numbers. Determine the missing term (?) from the following ratio.

$$? = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

A. outcomes

(B.) probability

C. population

D. sample

(3) 6. A sewing bag contains 56 yellow spools and 40 white spools of thread. If you randomly select a spool from the bag, what is the probability that the spool will **not** be white?

$$P(\textit{not white}) = \frac{56}{96}$$
$$= \frac{7}{12} \text{ or } 0.58\dot{3}$$

The probability that the spool of thread will not be white is 7 out of 12 or $0.58\dot{3}$.

(3) 7. The probability of rolling a sum of 3 with two dice is $P(\textit{sum of 3}) = \frac{1}{18}$.

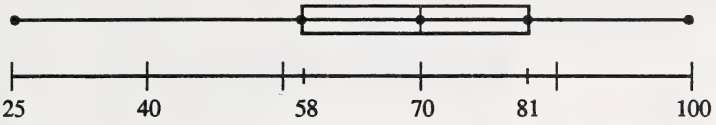
Given that a pair of dice is rolled 72 times, how many sums of 3 would you expect to occur?

$$\frac{\text{one sum of 3}}{\text{18 rolls}} = \frac{x}{\text{72 rolls}}$$
$$x = 72 \div 18$$
$$= 4$$

You would expect four sums of 3 to occur.

(3) 8. The following box-and-whisker plot shows the distribution of marks on a recent mathematics exam.

What is the probability that the mark for a student chosen at random is between 58% and 70%?




Note that 58% is the lower quartile mark and
70% is the median, so 25% or $\frac{1}{4}$ of all marks are in this area.

$\therefore P(58\% \text{ to } 70\%) = \frac{1}{4}$ or 25%

The probability that a student's mark is between 58% and 70% is 1 in 4.

**Topic 4**

__25__ marks

# Final Test

There are two copies of the final test: the teacher's copy which includes a marking guide, and the student's copy which is designed for photocopying and possible faxing.

**Note:**

The student's copy and the teacher's copy of this final test should be kept secure by the teacher.
Students should not have access to this test until it is assigned in a supervised situation.
The answers should be stored securely and retained by the teacher at all times.

# MATHEMATICS 10

## FINAL TEST

## GENERAL INSTRUCTIONS

1. The time limit for this test is **two hours**.

2. This test consists of two parts. Complete both parts.

3. Write all answers legibly in pen or pencil.

4. A scientific calculator is required to complete this test.

5. For written response questions, be sure to show all your work. Part marks will be given for partially correct solutions.

6. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

TOTAL MARKS: 100

PART A: Multiple Choice – 40 marks

PART B: Written Response – 60 marks

**TEACHER'S COPY**

**Value**

## PART A: MULTIPLE CHOICE

**40**

In each of the questions, indicate the correct answer by placing the appropriate letter A, B, C, or D in the blank provided.

Example:

__B__ The value of $(2-5)$ is

A. 3

B. $-3$

C. 7

D. $-7$

__D__ 1. Which of the following is a real number?

A. $-9+5i$

B. $3-3.9i$

C. $-4-3i$

D. 90.06

__A__ 2. What is the name given to 5 in the expression $\sqrt[4]{5}$?

A. radicand

B. index

C. radical

D. radical sign

A 3. Change $i\sqrt{39}$ to a form where you have the square root of a negative number. This form must only have a radicand.

A. $\sqrt{-39}$

B. $\sqrt{39}$

C. $\sqrt{-39i}$

D. $\sqrt{39i}$

A 4. Evaluate $\dfrac{-9+12(-3)}{24\div(-8)\times 3}$.

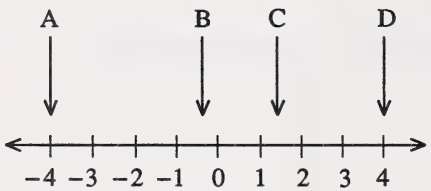
A. 5

B. 9

C. 1

D. 45

B 5. Locate $-0.4$ on the number line.



B 6. What degree is the polynomial $-6x^2yz^2 - 3xyz + 5y^4$?

A. 4

B. 5

C. 6

D. 12

C 7. Which statement is **not** correct?

A. $2^{-7} = \left(\frac{1}{2}\right)^7$

B. $\left(\frac{5}{8}\right)^{-2} = \left(\frac{8}{5}\right)^2$

C. $(-3)^{-3} = \left(\frac{1}{3}\right)^3$

D. $\left(-\frac{1}{4}\right)^{-5} = (-4)^5$

D 8. An example of a cubic trinomial is

A. $x^3 + y$

B. $9x^3$

C. $2xy^2z + 7x$

D. $-5xy + 7xyz + z^2$

B 9. $2xy^3\left(-3x^2y^2\right)=$

A. $-6x^2y^5$

B. $-6x^3y^5$

C. $-6x^2y^2$

D. $-x^3y^5$

B 10. What is $\frac{7^8\times7^9}{7^6}$ expressed as a single power?

A. $7^{12}$

B. $7^{11}$

C. $7^7$

D. $7^5$

A 11. The simplifed form of $\left(-5\sqrt{5}\right)^2$ is

A. 125

B. $-125$

C. $-25$

D. 25

A 12. Solve the inequality $\frac{x}{3} > \frac{3x}{5} - 4$.

A. $x < 15$

B. $x > 15$

C. $x < -15$

D. $x > -15$

C 13. Which expression means **four times a number decreased by five is less than seven**?

A. $4 \times n = 7 - 5$

B. $4n + 5 < 7$

C. $4n - 5 < 7$

D. $4 \times (n - 5) < 7$

D 14. There are 37 animals, either ducks or cows, in a field. There are 106 legs counted. How many cows are there?

A. 21

B. 32

C. 42

D. 16

C 15. The greatest common factor of $36x^2y$, $48x^2$, and $60x^2y^2$ is

A. $12x^2y^2$

B. $720x^2y^2$

C. $12x^2$

D. $60x^2y^2$

A 16. Factor $25x^2-30x+9$.

A. $(5x-3)(5x-3)$

B. $(25x+1)(x+9)$

C. $(5x+3)(5x-3)$

D. $(5x+3)(5x+3)$

A 17. Which binomial is a factor of $xy+x+y^2+y$?

A. $y+1$

B. $x+1$

C. $xy+x$

D. $x+y^2$

__C__ 18. The height of a triangle is 5 cm less than the base. If the area of the triangle is 18 $cm^2$, find the height.

A. 9 cm

B. 3.6 cm

C. 4 cm

D. 1.8 cm

__A__ 19. The factors of $x^2y^2 - 16$ are

A. $(xy-4)(xy+4)$

B. $(x)(y)(-4)$

C. $(x-4)(y+4)$

D. $(xy-4)(xy-4)$

__C__ 20. Two numbers differ by five. The sum of the squares of the numbers is 73. Find the numbers.

A. 8, 3

B. $-8, -3$

C. Both A and B

D. Neither A nor B

D 21. In which quadrant in the coordinate plane would you find the ordered pair $(2, -13)$?

A. Quadrant I

B. Quadrant II

C. Quadrant III

D. Quadrant IV

D 22. If $N(-8, -6)$ and $M(7, -10)$ are the endpoints of a line segment, then the coordinates of the midpoint of $\overline{NM}$ are

A. $\left(-7\frac{1}{2}, 2\right)$

B. $(-1, 4)$

C. $\left(-7, -1\frac{1}{2}\right)$

D. $\left(-\frac{1}{2}, -8\right)$

B 23. The coordinates of the endpoints of the diameter of a circle are $(-4, -4)$ and $(10, 7)$. Find the length of the radius (rounded to the nearest tenth).

A. 3.0 units

B. 8.9 units

C. 1.5 units

D. 4.5 units

__B__ 24. The slope of the line joining the points $(3, 19)$ and $(4, 23)$ is

A. $\frac{1}{4}$

B. 4

C. $\frac{3}{4}$

D. $\frac{4}{3}$

__B__ 25. Which of the following statements is **not** true?

A. The slope of a horizontal line is zero.

B. A line that has a positive slope will fall to the right.

C. The slope of a vertical line is undefined.

D. A line that has a negative slope will fall to the right.

__D__ 26. Read the following statements.

Statement 1: If two lines are perpendicular, the product of their slope is $-1$.

Statement 2: Parallel lines have equal slopes.

Statement 3: Collinear points lie on the same straight line.

Which statements are true?

A. Statements 1 and 2

B. Statements 1 and 3

C. Statements 2 and 3

D. Statements 1, 2, and 3

A 27. What is the $y$-intercept of $2x+3y-6=0$?

A. 2

B. 3

C. $-2$

D. $-3$

C 28. What is the classification of the following system of equations?

$-x+2y=10$
$8x=16y$

A. dependent

B. independent

C. inconsistent

D. none of the above

B 29. The slope $y$-intercept form of the equation $2x-3y=5$ is

A. $-3y=5-2x$

B. $y=\frac{2}{3}x-\frac{5}{3}$

C. $y=\frac{2}{3}y+\frac{5}{3}$

D. $y=-\frac{2}{3}x+5$

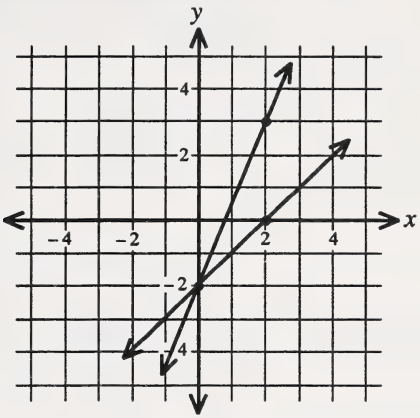
__A__ 30. Which system of equations is shown on the graph?

A. $x-y=2$
$5x-2y=4$

B. $x-3y=6$
$2x-6y=12$

C. $2x+3y=-6$
$x+y=-2$

D. $x-y=2$
$2x-5y=6$




__D__ 31. What is the solution to the following system of equations?

$2x+y=12$
$2x-3y=4$

A. $(2,5)$

B. $(4,4)$

C. $(8,4)$

D. $(5,2)$

D 32. Yena invested \$5000, part at 9% per annum and the remainder at 11%. One year later the interest earned on the 9% investment was \$350 less than the interest earned on the 11% investment. Which system of equations shows the amount invested at each rate?

A. $x+y=5000$
$0.11x+0.09y=350$

B. $x-y=5000$
$0.11x-0.09y=350$

C. $x-y=5000$
$0.11x+0.09y=350$

D. $x+y=5000$
$0.11x-0.09y=350$

D 33. In $\Delta PQR$, $\angle P=90°$. Write the tangent ratio of $\angle Q$.

A. $\frac{q}{p}$

B. $\frac{r}{p}$

C. $\frac{p}{q}$

D. $\frac{q}{r}$

C 34. The diagonal of a rectangle is 12.5 m long. The width of the rectangle is 6.1 m. Calculate the length of the rectangle rounded to one decimal place.

A. 13.9 m

B. 10.8 m

C. 10.9 m

D. 14.0 m

A 35. A flag pole casts a shadow 28 m long. At the same time, a tree 2 m tall casts a shadow that is 3.5 m long. Find the height of the flagpole to the nearest metre.

A. 16 m

B. 49 m

C. 29 m

D. 26 m

D 36. $ABC$ is a right triangle with the right angle at $B$. If $\angle C$ measures 77°, find tan $A$. Round to four decimal places.

A. 0.9744

B. 4.3315

C. 4.331

D. 0.2309

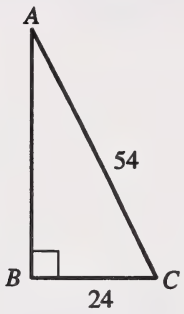
B 37. In $\Delta ABC$, find $\angle A$ to the nearest degree.

A. 44°

B. 26°

C. 64°

D. 24°




A 38. A card is drawn from a well-shuffled deck of 52 cards. If there are two red aces in the deck of cards, the probability of drawing a red ace is

A. $\frac{1}{26}$

B. $\frac{1}{13}$

C. $\frac{1}{52}$

D. $\frac{2}{13}$

Use the following stem-and-leaf plot to answer questions 39 and 40.

RBI totals for 35 Distance League Ball Players

| | |
|---|---|
| 10 | 5 6 9 9 |
| 11 | 1 8 9 9 |
| 12 | 0 0 1 1 1 1 1 5 6 6 6 7 7 9 9 |
| 13 | 0 0 2 6 7 |
| 14 | 1 2 2 3 8 9 |
| 15 | 3 |

_C_ 39. What is the mode of the data?

A. 9

B. 1

C. 121

D. 126

_D_ 40. What is the median of the data?

A. 9

B. 1

C. 121

D. 126

**Part A**

40 marks

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**Value**

## PART B: WRITTEN RESPONSE QUESTIONS

**60**

Be sure to show all your work. Partial marks will be assigned for any correct work shown even if the final answer is incorrect.

1. Perform the required operation for the following, and express the answer in simplified form.

(1) a. $2\frac{1}{7} \div (-3) = 2\frac{1}{7} \div \left(-\frac{3}{1}\right)$

$= \frac{15}{7} \times \left(-\frac{1}{3}\right)$

$= -\frac{15}{21}$

$= -\frac{5}{7}$

(1) b. $(0.02 \times 0.004) \div 0.8 = 0.00008 \div 0.8$

$= 0.0001$

(1) 2. Convert 3.12 to a fractional number in lowest terms.

$3.12 = \frac{312}{100}$

$= 3\frac{12}{100}$

$= 3\frac{3}{25}$

(1) 3. 63 is 7% of what number?

$$\frac{63}{x} = \frac{7}{100}$$
$$7x = 6300$$
$$x = \frac{6300}{7}$$
$$= 900$$

Therefore, 63 is 7% of 900.

(1) 4. Six less than twice a certain number is more than 22. What range of values is possible for the number?

$$2x - 6 > 22$$
$$2x > 28$$
$$x > 14$$

Any value greater than 14 is a possible value for this number.

(1) 5. Convert $\sqrt{363}$ to a mixed radical.

$$\sqrt{363} = \sqrt{3 \times 11 \times 11}$$
$$= 11\sqrt{3}$$

(2) 6. Find the value of $\frac{2x^3-3xy^2}{y}$ when $x=-1$ and $y=2$.

$$\frac{2x^3-3xy^2}{y}=\frac{2(-1)^3-3(-1)(2)^2}{2}$$
$$=\frac{-2+12}{2}$$
$$=\frac{10}{2}$$
$$=5$$

(1) 7. Simplify $\frac{-4xy^3}{-16x^2y}$.

$$\frac{-4xy^3}{-16x^2y}=\frac{4xy^3}{16x^2y}$$
$$=\frac{1y^{3-1}}{4x^{2-1}}$$
$$=\frac{y^2}{4x}$$

(2) 8. Simplify $(m+n)^2+3(mn-2n^2)$.

$$(m+n)^2+3(mn-2n^2)=m^2+2mn+n^2+3mn-6n^2$$
$$=m^2+5mn-5n^2$$

**TEACHER'S COPY**

(2) 9. Divide $12y^2 - 13y - 30$ by $3y - 7$ using the long division method.

$$
\begin{array}{r}
4y+5 \\
3y-7\overline{)12y^2-13y-30} \\
\underline{12y^2-28y} \\
15y-30 \\
\underline{15y-35} \\
5
\end{array}
$$

(3) 10. Solve for $y$ in $\frac{y}{6}+\frac{1}{4}(y+5)=\frac{1}{3}(y+8)$.

(The L.C.D. is 12.)

$$\frac{y}{6}+\frac{1}{4}(y+5)=\frac{1}{3}(y+8)$$

$$12\left(\frac{y}{6}\right)+12\left(\frac{y}{4}\right)+12\left(\frac{5}{4}\right)=12\left(\frac{y}{3}\right)+12\left(\frac{8}{3}\right)$$

$$2y+3y+15=4y+32$$

$$5y+15=4y+32$$

$$5y-4y=32-15$$

$$y=17$$

(2) 11. Solve for $x$ in $\sqrt{4(x-5)}=4$.

$$\sqrt{4(x-5)}=4$$
$$\left(\sqrt{4(x-5)}\right)^2=(4)^2$$
$$4(x-5)=16$$
$$4x-20=16$$
$$4x=16+20$$
$$4x=36$$
$$\frac{4x}{4}=\frac{36}{4}$$
$$x=9$$

(2) 12. Factor the polynomial $25-(a-b)^2$.

$$25-(a-b)^2=[5-(a-b)][5+(a-b)]$$
$$=(5-a+b)(5+a-b)$$

(2) 13. Find the zeros of $2x^2-7x-4$.

$$2x^2-7x-4=(2x+1)(x-4)$$

$2x+1=0$ or $x-4=0$

$2x=-1$ $\quad x=4$

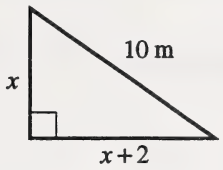
$x=-\frac{1}{2}$

The zeros are $-\frac{1}{2}$ and 4.

**TEACHER'S COPY**

(3) 14. The length of a rectangular metal sheet is 2 m more than its width. If the measure of each diagonal is 10 m, find the dimensions of the sheet.

Let $x$ be the width.
Thus, the length is $(x+2)$.




$$x^2+(x+2)^2=10^2$$
$$x^2+x^2+4x+4=100$$
$$2x^2+4x-96=0$$
$$x^2+2x-48=0$$
$$(x-6)(x+8)=0$$

$x-6=0$ or $x+8=0$
$x=6$ $\qquad$ $x=-8$

The solution $x=-8$ is rejected since length cannot be negative.

Therefore, the width of the rectangle is 6 m and the length is $6+2=8$ m.

(2) 15. The midpoint of a line segment is $(4,16)$. One endpoint of the line segment is $(8,22)$. What is the other endpoint?

$$\frac{x_1+x_2}{2}=4 \qquad \frac{y_1+y_2}{2}=16$$
$$\frac{8+x_2}{2}=4 \qquad \frac{22+y_2}{2}=16$$
$$8+x_2=8 \qquad 22+y_2=32$$
$$x_2=8-8 \qquad y_2=32-22$$
$$=0 \qquad =10$$

The other endpoint of the line segment is $(0,10)$.

(3) 16. Find the value of $x$ so that the line passing through $A(x,-2)$ and $B(-1,2)$ is perpendicular to the line passing through $P(4,3)$ and $Q(3,-2)$.

$$m_{\overline{AB}} = \frac{y_1 - y_2}{x_1 - x_2} \qquad m_{\overline{PQ}} = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{-2-2}{x+1} \qquad = \frac{3-(-2)}{4-3}$$
$$= \frac{-4}{x+1} \qquad = \frac{3+2}{1}$$
$$= \frac{5}{1}$$

Since the slopes of perpendicular lines are negative reciprocals, change $\frac{5}{1}$ to $\frac{-1}{5}$. Now solve for $x$.

$$\frac{-4}{x+1} = \frac{-1}{5}$$
$$-1(x+1) = -4(5)$$
$$-x-1 = -20$$
$$-x = -20+1$$
$$-x = -19$$
$$x = 19$$

The value of $x$ in $A(x,-2)$ is 19.

③

17. Determine the equation of the line passing through the points $(4,-8)$ and $(-2,-12)$. Express your equation in the form $Ax+By+C=0$, where $a$ is positive.

Find the slope using $m=\dfrac{y_1-y_2}{x_1-x_2}$; then use the point-slope formula which is $y-y_1=m(x-x_1)$.

$$\begin{aligned} m &= \frac{y_1-y_2}{x_1-x_2} \\ &= \frac{-8-(-12)}{4-(-2)} \\ &= \frac{-8+12}{4+2} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-12) &= \frac{2}{3}[x-(-2)] \\ y+12 &= \frac{2}{3}(x+2) \quad \text{(Multiply by 3.)} \\ 3(y+12) &= 2(x+2) \\ 3y+36 &= 2x+4 \\ 3y-2x+36-4 &= 0 \\ -2x+3y+32 &= 0 \\ 2x-3y-32 &= 0 \end{aligned}$$

The equation is $2x-3y-32=0$.

**TEACHER'S COPY**

(3)

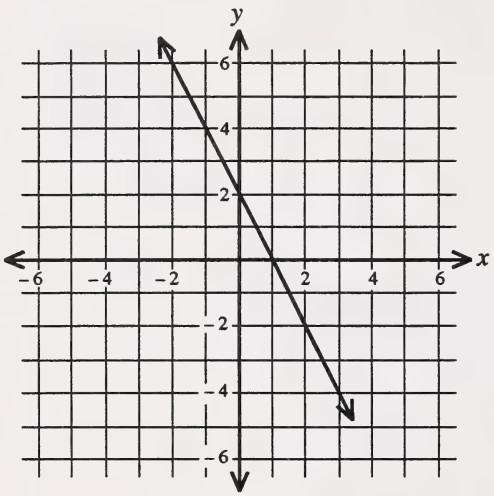
18. Determine the slope of the line described by $2x+y=2$. Sketch the graph of the line using the slope and the $y$-intercept.

The slope is as follows:

$$2x+y=2$$
$$y=-2x+2$$
$$\therefore m=-2$$

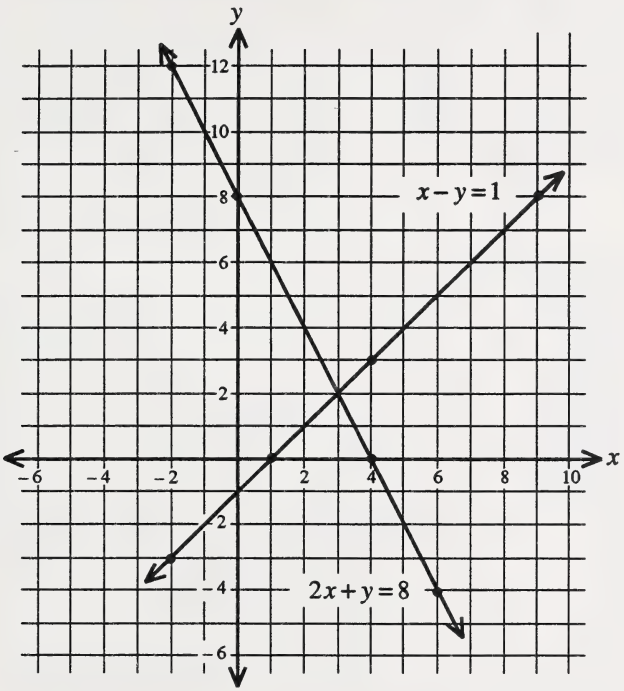
The $y$-intercept is as follows:

$$2x+y=2$$
$$2(0)+y=2$$
$$y=2$$



④

19. A tracking ship is on a course given by $2x + y = 8$. The ship has predicted the course of a submarine to be $x - y = 1$. Determine the coordinates of the point where their paths will cross.




The two paths will intersect at point $(3, 2)$.

**TEACHER'S COPY**

(3) 20. Solve the following system of equations.

$$\begin{cases} 2y-5x=1 & (1) \\ x=-y-3 & (2) \end{cases}$$

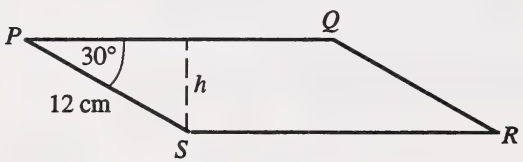
$$\begin{aligned} & y+x=-3 && (3) \\ (3)\times 2: \quad & 2y+2x=-6 && (4) \\ & 2y-5x=1 && (1) \\ \hline (4)-(1): \quad & 7x=-7 \\ & x=-1 \end{aligned}$$

Substitute $x=-1$ in (3).

$$\begin{aligned} y-1&=-3 \\ y&=-2 \end{aligned}$$

$$\therefore (x, y)=(-1,-2)$$

(3) 21. In the parallelogram $PQRS$, find the altitude $h$ if $PS=12$ cm.




$$\begin{aligned} \frac{h}{12} &= \sin 30^\circ \\ h &= 12 \sin 30^\circ \\ &= 12(0.5) \\ &= 6 \end{aligned}$$

The altitude $h$ is 6 cm in length.

**TEACHER'S COPY**

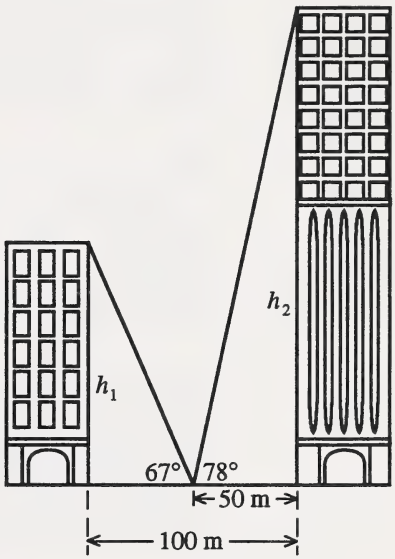
⑤

22. Two office towers are 100 m apart. An observer halfway between finds the measure of the angle of elevation to the top of the shorter tower to be 67° and the measure of the angle of elevation to the top of the other tower to be 78°. Find the difference in height of the two towers. Round your answer to the nearest metre.

$$\tan 67° = \frac{h_1}{50}$$
$$h_1 = 50 \tan 67°$$
$$\doteq 50(2.355\,852\,366)$$
$$\doteq 117.792\,6183$$

$$\tan 78° = \frac{h_2}{50}$$
$$h_2 = 50 \tan 78°$$
$$\doteq 50(4.704\,630\,11)$$
$$\doteq 235.231\,5055$$

$$\text{difference} \doteq 235.231\,5055 - 117.792\,6183$$
$$\doteq 117.438\,8872$$
$$\doteq 117$$




The difference in the height of the two towers is approximately 117 m.

(2) 23. Suppose the probability of selecting a yellow button from a box of coloured buttons is $P(yellow) = \frac{1}{13}$. If there is a total of 91 buttons in the box, how many yellow buttons would there be?

$\frac{1}{13} \times 91 = 7$

There would be 7 yellow buttons.

(3) 24. Suppose you want to find the average number of hours a Canadian high school student watches television. Explain how the sample should be picked. Give reasons why you would pick the sample this way.

The sample should include both rural and urban students from schools across the entire country. Both rural and urban students should be included because students who live in cities may not watch the same amount of television as students who live in the country. A cross section of the country is required because students in some parts may have only television as their main source of entertainment, while students in other parts may not have access to television.

25. The following is the total number of points scored by the Golden Raiders in an 18-game system.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 56 | 50 | 44 | 54 | 52 | 50 | 40 | 58 | 36 |
| 50 | 40 | 44 | 34 | 48 | 56 | 52 | 36 | 42 |

(2) a. Find the median, the lower quartile, the upper quartile, and the range.

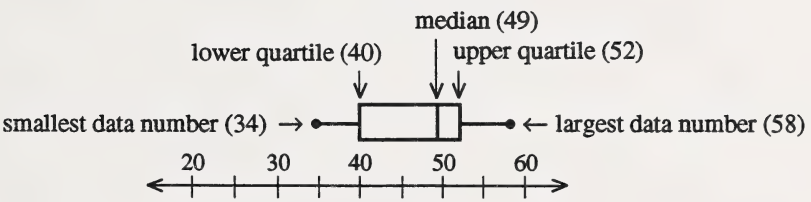
Rearrange the data in accending order.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 34 | 36 | 36 | 40 | **40** | 42 | 44 | 44 | **48** |
| **50** | 50 | 50 | 52 | **52** | 54 | 56 | 56 | 58 |

$$\text{median} = \frac{48+50}{2} \qquad \text{lower quartile} = 40$$
$$= \frac{98}{2}$$
$$= 49$$

$$\text{upper quartile} = 52 \qquad \text{range} = 58 - 34$$
$$= 24$$

(2) b. Construct a box-and-whisker plot.




**Part B**

__60__ marks

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# MATHEMATICS 10

## FINAL TEST

## GENERAL INSTRUCTIONS

1. The time limit for this test is **two hours**.

2. This test consists of two parts. Complete both parts.

3. Write all answers legibly in pen or pencil.

4. A scientific calculator is required to complete this test.

5. For written response questions, be sure to show all your work. Part marks will be given for partially correct solutions.

6. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

TOTAL MARKS: 100

PART A: Multiple Choice – 40 marks

PART B: Written Response – 60 marks

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

**Value**

**40**

## PART A: MULTIPLE CHOICE

In each of the questions, indicate the correct answer by placing the appropriate letter A, B, C, or D in the blank provided.

Example:

*B* The value of $(2-5)$ is

A. 3

B. $-3$

C. 7

D. $-7$

_____ 1. Which of the following is a real number?

A. $-9+5i$

B. $3-3.9i$

C. $-4-3i$

D. 90.06

_____ 2. What is the name given to 5 in the expression $\sqrt[4]{5}$?

A. radicand

B. index

C. radical

D. radical sign

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 3. Change $i\sqrt{39}$ to a form where you have the square root of a negative number. This form must only have a radicand.

A. $\sqrt{-39}$

B. $\sqrt{39}$

C. $\sqrt{-39i}$

D. $\sqrt{39i}$

_____ 4. Evaluate $\dfrac{-9+12(-3)}{24\div(-8)\times 3}$.

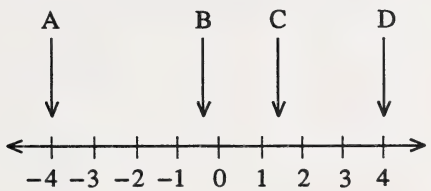
A. 5

B. 9

C. 1

D. 45

_____ 5. Locate $-0.4$ on the number line.




Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 6. What degree is the polynomial $-6x^2yz^2 - 3xyz + 5y^4$?

A. 4

B. 5

C. 6

D. 12

_____ 7. Which statement is **not** correct?

A. $2^{-7} = \left(\frac{1}{2}\right)^7$

B. $\left(\frac{5}{8}\right)^{-2} = \left(\frac{8}{5}\right)^2$

C. $(-3)^{-3} = \left(\frac{1}{3}\right)^3$

D. $\left(-\frac{1}{4}\right)^{-5} = (-4)^5$

_____ 8. An example of a cubic trinomial is

A. $x^3 + y$

B. $9x^3$

C. $2xy^2z + 7x$

D. $-5xy + 7xyz + z^2$

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 9. $2xy^3\left(-3x^2y^2\right)=$

A. $-6x^2y^5$

B. $-6x^3y^5$

C. $-6x^2y^2$

D. $-x^3y^5$

_____ 10. What is $\frac{7^8\times7^9}{7^6}$ expressed as a single power?

A. $7^{12}$

B. $7^{11}$

C. $7^7$

D. $7^5$

_____ 11. The simplifed form of $\left(-5\sqrt{5}\right)^2$ is

A. 125

B. $-125$

C. $-25$

D. 25

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 12. Solve the inequality $\frac{x}{3} > \frac{3x}{5} - 4$.

A. $x < 15$

B. $x > 15$

C. $x < -15$

D. $x > -15$

_____ 13. Which expression means **four times a number decreased by five is less than seven**?

A. $4 \times n = 7 - 5$

B. $4n + 5 < 7$

C. $4n - 5 < 7$

D. $4 \times (n - 5) < 7$

_____ 14. There are 37 animals, either ducks or cows, in a field. There are 106 legs counted. How many cows are there?

A. 21

B. 32

C. 42

D. 16

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_____ 15. The greatest common factor of $36x^2y$, $48x^2$, and $60x^2y^2$ is

A. $12x^2y^2$

B. $720x^2y^2$

C. $12x^2$

D. $60x^2y^2$

_____ 16. Factor $25x^2 - 30x + 9$.

A. $(5x-3)(5x-3)$

B. $(25x+1)(x+9)$

C. $(5x+3)(5x-3)$

D. $(5x+3)(5x+3)$

_____ 17. Which binomial is a factor of $xy + x + y^2 + y$?

A. $y+1$

B. $x+1$

C. $xy+x$

D. $x+y^2$

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_____ 18. The height of a triangle is 5 cm less than the base. If the area of the triangle is $18\ cm^2$, find the height.

A. 9 cm

B. 3.6 cm

C. 4 cm

D. 1.8 cm

_____ 19. The factors of $x^2y^2 - 16$ are

A. $(xy-4)(xy+4)$

B. $(x)(y)(-4)$

C. $(x-4)(y+4)$

D. $(xy-4)(xy-4)$

_____ 20. Two numbers differ by five. The sum of the squares of the numbers is 73. Find the numbers.

A. 8, 3

B. $-8, -3$

C. Both A and B

D. Neither A nor B

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_____ 21. In which quadrant in the coordinate plane would you find the ordered pair $(2, -13)$?

A. Quadrant I

B. Quadrant II

C. Quadrant III

D. Quadrant IV

_____ 22. If $N(-8, -6)$ and $M(7, -10)$ are the endpoints of a line segment, then the coordinates of the midpoint of $\overline{NM}$ are

A. $\left(-7\frac{1}{2}, 2\right)$

B. $(-1, 4)$

C. $\left(-7, -1\frac{1}{2}\right)$

D. $\left(-\frac{1}{2}, -8\right)$

_____ 23. The coordinates of the endpoints of the diameter of a circle are $(-4, -4)$ and $(10, 7)$. Find the length of the radius (rounded to the nearest tenth).

A. 3.0 units

B. 8.9 units

C. 1.5 units

D. 4.5 units

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_____ 24. The slope of the line joining the points $(3, 19)$ and $(4, 23)$ is

A. $\frac{1}{4}$

B. 4

C. $\frac{3}{4}$

D. $\frac{4}{3}$

_____ 25. Which of the following statements is **not** true?

A. The slope of a horizontal line is zero.

B. A line that has a positive slope will fall to the right.

C. The slope of a vertical line is undefined.

D. A line that has a negative slope will fall to the right.

_____ 26. Read the following statements.

Statement 1: If two lines are perpendicular, the product of their slope is $-1$.

Statement 2: Parallel lines have equal slopes.

Statement 3: Collinear points lie on the same straight line.

Which statements are true?

A. Statements 1 and 2

B. Statements 1 and 3

C. Statements 2 and 3

D. Statements 1, 2, and 3

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_____ 27. What is the $y$-intercept of $2x+3y-6=0$?

A. 2

B. 3

C. $-2$

D. $-3$

_____ 28. What is the classification of the following system of equations?

$-x+2y=10$
$8x=16y$

A. dependent

B. independent

C. inconsistent

D. none of the above

_____ 29. The slope $y$-intercept form of the equation $2x-3y=5$ is

A. $-3y=5-2x$

B. $y=\frac{2}{3}x-\frac{5}{3}$

C. $y=\frac{2}{3}y+\frac{5}{3}$

D. $y=-\frac{2}{3}x+5$

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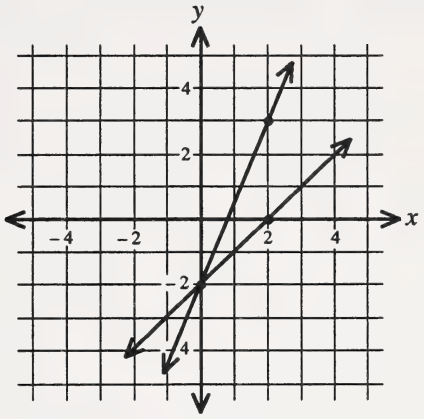
_____ 30. Which system of equations is shown on the graph?

A. $x-y=2$
$5x-2y=4$

B. $x-3y=6$
$2x-6y=12$

C. $2x+3y=-6$
$x+y=-2$

D. $x-y=2$
$2x-5y=6$




_____ 31. What is the solution to the following system of equations?

$2x+y=12$
$2x-3y=4$

A. $(2,5)$

B. $(4,4)$

C. $(8,4)$

D. $(5,2)$

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_____ 32. Yena invested \$5000, part at 9% per annum and the remainder at 11%. One year later the interest earned on the 9% investment was \$350 less than the interest earned on the 11% investment. Which system of equations shows the amount invested at each rate?

A. $x+y=5000$
$0.11x+0.09y=350$

B. $x-y=5000$
$0.11x-0.09y=350$

C. $x-y=5000$
$0.11x+0.09y=350$

D. $x+y=5000$
$0.11x-0.09y=350$

_____ 33. In $\Delta PQR$, $\angle P=90°$. Write the tangent ratio of $\angle Q$.

A. $\frac{q}{p}$

B. $\frac{r}{p}$

C. $\frac{p}{q}$

D. $\frac{q}{r}$

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_____ 34. The diagonal of a rectangle is 12.5 m long. The width of the rectangle is 6.1 m. Calculate the length of the rectangle rounded to one decimal place.

A. 13.9 m

B. 10.8 m

C. 10.9 m

D. 14.0 m

_____ 35. A flag pole casts a shadow 28 m long. At the same time, a tree 2 m tall casts a shadow that is 3.5 m long. Find the height of the flagpole to the nearest metre.

A. 16 m

B. 49 m

C. 29 m

D. 26 m

_____ 36. $ABC$ is a right triangle with the right angle at $B$. If $\angle C$ measures 77°, find tan $A$. Round to four decimal places.

A. 0.9744

B. 4.3315

C. 4.331

D. 0.2309

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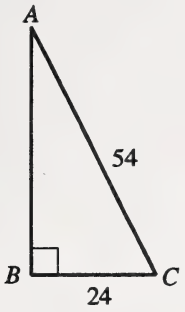
_____ 37. In $\Delta ABC$, find $\angle A$ to the nearest degree.

A. 44°

B. 26°

C. 64°

D. 24°




_____ 38. A card is drawn from a well-shuffled deck of 52 cards. If there are two red aces in the deck of cards, the probability of drawing a red ace is

A. $\frac{1}{26}$

B. $\frac{1}{13}$

C. $\frac{1}{52}$

D. $\frac{2}{13}$

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Use the following stem-and-leaf plot to answer questions 39 and 40.

RBI totals for 35 Distance League Ball Players

| | |
|---|---|
| 10 | 5 6 9 9 |
| 11 | 1 8 9 9 |
| 12 | 0 0 1 1 1 1 1 5 6 6 6 7 7 9 9 |
| 13 | 0 0 2 6 7 |
| 14 | 1 2 2 3 8 9 |
| 15 | 3 |

_____ 39. What is the mode of the data?

A. 9

B. 1

C. 121

D. 126

_____ 40. What is the median of the data?

A. 9

B. 1

C. 121

D. 126

**Part A**

_____ marks

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**Value**

**60**

## PART B: WRITTEN RESPONSE QUESTIONS

Be sure to show all your work. Partial marks will be assigned for any correct work shown even if the final answer is incorrect.

1. Perform the required operation for the following, and express the answer in simplified form.

(1) a. $2\frac{1}{7} \div (-3)$

(1) b. $(0.02 \times 0.004) \div 0.8$

(1) 2. Convert 3.12 to a fractional number in lowest terms.

Name of Student _____ Student I.D. # _____

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(1) 3. 63 is 7% of what number?

(1) 4. Six less than twice a certain number is more than 22. What range of values is possible for the number?

(1) 5. Convert $\sqrt{363}$ to a mixed radical.

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(2) 6. Find the value of $\frac{2x^3-3xy^2}{y}$ when $x=-1$ and $y=2$.

(1) 7. Simplify $\frac{-4xy^3}{-16x^2y}$.

(2) 8. Simplify $(m+n)^2+3(mn-2n^2)$.

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(2) 9. Divide $12y^2 - 13y - 30$ by $3y - 7$ using the long division method.

(3) 10. Solve for $y$ in $\frac{y}{6} + \frac{1}{4}(y+5) = \frac{1}{3}(y+8)$.

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(2) 11. Solve for $x$ in $\sqrt{4(x-5)} = 4$.

(2) 12. Factor the polynomial $25-(a-b)^2$.

(2) 13. Find the zeros of $2x^2 - 7x - 4$.

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(3) 14. The length of a rectangular metal sheet is 2 m more than its width. If the measure of each diagonal is 10 m, find the dimensions of the sheet.

(2) 15. The midpoint of a line segment is $(4, 16)$. One endpoint of the line segment is $(8, 22)$. What is the other endpoint?

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(3) 16. Find the value of $x$ so that the line passing through $A(x, -2)$ and $B(-1, 2)$ is perpendicular to the line passing through $P(4, 3)$ and $Q(3, -2)$.

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(3) 17. Determine the equation of the line passing through the points $(4, -8)$ and $(-2, -12)$. Express your equation in the form $Ax + By + C = 0$, where $a$ is positive.

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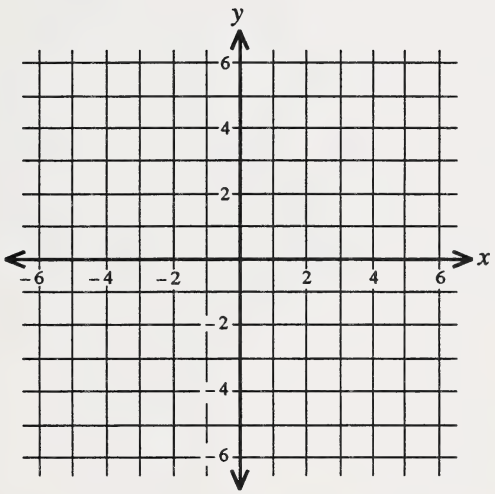
(3)

18. Determine the slope of the line described by $2x + y = 2$. Sketch the graph of the line using the slope and the $y$-intercept.

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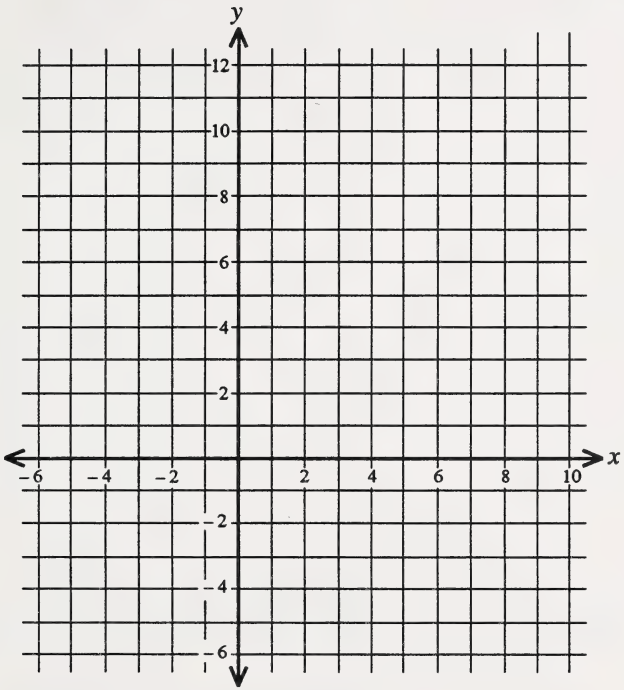
④

19. A tracking ship is on a course given by $2x + y = 8$. The ship has predicted the course of a submarine to be $x - y = 1$. Determine the coordinates of the point where their paths will cross.




Name of Student _____ Student I.D. # _____

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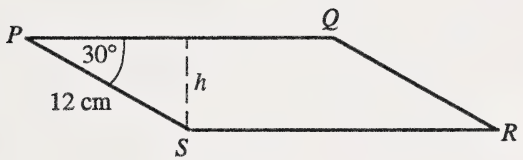
(3) 20. Solve the following system of equations.

$$\begin{cases} 2y-5x=1 \\ x=-y-3 \end{cases}$$

(3) 21. In the parallelogram $PQRS$, find the altitude $h$ if $PS=12$ cm.




Name of Student _____ Student I.D. # _____

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(5)

22. Two office towers are 100 m apart. An observer halfway between finds the measure of the angle of elevation to the top of the shorter tower to be 67° and the measure of the angle of elevation to the top of the other tower to be 78°. Find the difference in height of the two towers. Round your answer to the nearest metre.

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(2) 23. Suppose the probability of selecting a yellow button from a box of coloured buttons is $P(yellow) = \frac{1}{13}$. If there is a total of 91 buttons in the box, how many yellow buttons would there be?

(3) 24. Suppose you want to find the average number of hours a Canadian high school student watches television. Explain how the sample should be picked. Give reasons why you would pick the sample this way.

Name of Student _____ Student I.D. # _____

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25. The following is the total number of points scored by the Golden Raiders in an 18-game system.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 56 | 50 | 44 | 54 | 52 | 50 | 40 | 58 | 36 |
| 50 | 40 | 44 | 34 | 48 | 56 | 52 | 36 | 42 |

(2) a. Find the median, the lower quartile, the upper quartile, and the range.

Rearrange the data in accending order.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 34 | 36 | 36 | 40 | **40** | 42 | 44 | 44 | **48** |
| **50** | 50 | 50 | 52 | **52** | 54 | 56 | 56 | 58 |

(2) b. Construct a box-and-whisker plot.

**Part B**

_____ marks

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

# TEACHER QUESTIONNAIRE FOR MATHEMATICS 10

***This is a course designed in a new distance-learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated so that a future revision may incorporate any necessary improvements.***

Teacher's Name _____ Area of Expertise _____

School Name _____ Date _____

## Design

1. The units follow a definite systematic design. Did you find it easy to follow?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Did your observations reveal that the students found the design easy to follow?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you find the Learning Facilitator's Manual helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

4. Part of the design involves stating the objectives in student terms. Do you feel this helped the students understand what they were going to learn?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

5. The topics include self-assessment questions. These questions are to help clarify and reinforce the instructional materials. Did students use the answers effectively?

❑ Yes ❑ No If no, explain.

_____

_____

6. Did the Extra Help and Extensions prove to be helpful?

❑ Yes ❑ No If no, explain.

_____

_____

7. Were students motivated to try the Extra Help and Extensions?

❑ Yes ❑ No If no, give details.

_____

_____

8. Suggestions for computer, video, and audio activities are included in the course. Were your students able to use these activities?

❑ Yes ❑ No Comment on the lines below.

_____

_____

9. Were the assignments appropriate?

❑ Yes ❑ No If no, give details.

_____

_____

Teacher's Name _____

Name of School _____ Date _____

10. Did you fax assignments?

❑ Yes ❑ No

11. If you did fax, did you get satisfactory results from using this procedure?

❑ Yes ❑ No If no, give details.

_____

_____

## Instruction

1. Did you find the instruction clear?

❑ Yes ❑ No If no, give details.

_____

_____

2. Did your observations reveal that the students found the instruction interesting?

❑ Yes ❑ No If no, give details.

_____

_____

3. Did you find the instruction adequate?

❑ Yes ❑ No If no, give details.

_____

_____

4. Was the reading level appropriate?

❑ Yes ❑ No If no, give details.

_____

_____

Teacher's Name _____

Name of School _____ Date _____

5. Was the work load adequate?

❑ Yes ❑ No If no, give details.

_____

_____

6. Was the content accurate and current?

❑ Yes ❑ No If no, give details.

_____

_____

7. Did the content flow consistently and logically?

❑ Yes ❑ No If no, give details.

_____

_____

8. Was the transition between booklets smooth?

❑ Yes ❑ No If no, give details.

_____

_____

9. Was the transition between print and media smooth?

❑ Yes ❑ No If no, give details.

_____

_____

Teacher's Name _____

Name of School _____ Date _____

## Additional Comments

Thanks for taking the time to complete this survey. Your feedback is important to us.

Fax Number: 674-6686

Instructional Design and Development Unit
Alberta Distance Learning Centre
Box 4000
Barrhead, Alberta
T0G 2P0

Teacher's Name _____

Name of School _____ Date _____

Mathematics 10

Learning Facilitator's Manual

L.R.D.C.
Producer

1992